Modelling the dynamics of appliance price-efficiency distributions

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Abstract
In this study, we develop a novel modelling framework that may be useful in forecasting evolution of efficiency distributions and price-efficiency correlations seen in efficient appliance markets. The framework is founded on three basic assumptions: (1) technological learning is the primary driver of price changes for products at fixed efficiency, (2) the dynamics of product market distributions can be accurately characterized by simple dynamic cumulative distribution functions, and (3) price distributions at any particular time can be characterized by a 'willingness-to-pay" price distribution that is correlated with efficiency and which can be interpolated between a top-of-market value, and a bottom of market value. We combined these three fundamental assumptions with the mathematics of partial differential equations to derive a set of equations that describe the evolution of price and efficiency as a function of time and as a function of a transformed market share variable. The transformed market share variable quantifies the market position of each efficiency performance level relative to other efficiency performance levels. The key driver for the change in price at different efficiency levels is the increase of a "cumulative experience" variable that reflects the technological learning that occurs at and above each efficiency performance level. We demonstrate that the model captures the basic empirical observations regarding the evolution of efficiency distributions and price-efficiency functions seen in efficient refrigeration appliance markets in Europe between 1995 and 2009.

Introduction
In this study we develop a mathematical model for the dynamics of price-efficiency distributions for appliances. Because markets are dynamic, both appliance price, appliance efficiency, and the correlation between price and efficiency changes over time. The model that we develop incorporates technological learning-by-doing into a mathematical forecast of the time evolution of the distribution price as a function of efficiency. A key goal of this research is the development of price-efficiency forecasting methods that can be useful in energy efficiency policy impact analyses.

Energy efficiency is widely viewed as a key element of an energy policy that can help mitigate climate change by decreasing the dependence on fossil fuels while simultaneously creating economic savings and benefits to consumers and the economy[1, 2, 3]. The vast majority of policy impact studies for appliance energy efficiency policies in the US have assumed that the incremental costs of efficiency increase with inflation or decline relatively slowly over time[4, 5, 6]. While some recent work has identified and quantified learning curve trends in the base price of appliances that can have a significant impact on the long term trajectory of the price of efficient appliances[7, 8], retrospective studies have had difficulty detecting the increases in average appliance price that appear to be projected by policy impact forecasts[9, 10, 11]. Some recent studies and discussions have suggested that the incremental price of efficiency may decrease at rates that are substantially faster than the rate of decline of average appliance price[12]. The apparently rapid rate of decline in the incremental price of efficiency may be related to technological learning.

This study provides a candidate series of equations that may provide a starting point for modeling dynamic price-efficiency curves that change over time due to technological innovation and learning.

In what follows, we first review some basic modeling principals and assumptions that we use to guide the development of our model equations. We then review some of the observed features of price, efficiency, and adoption in the European residential refrigerator market to motivate the basic structure of the model equations. We proceed next with the formulation of a proposed set of model equations that model the time evolution of the price-efficiency distribution consistent with our model development strategy. We then illustrate how the solutions to the partial differential equations that we develop approximate the observed features of the European residential refrigerator market. Finally we conclude with a discussion of potentially fruitful areas of future research.

Model Development Strategy

Before we launch in to the details of the model equations, we first review a set of strategic modeling choices that we have made to guide our model development. We first list these key modeling choices, and then briefly describe why we have made each of these choices. The key assumptions of our model equations are as follows:

1. *Learning-by-doing dominates price dynamics:* The cost of production for a product, or a piece of a product obeys the dynamics of "learning by doing" and follows the dynamics a learning curve.
2. *The model forecasts the dynamics of cumulative market distributions:* We specify the quantity of interest for forecasting purposes to be the cumulative price-efficiency distribution which is the fraction of product sales at or above a particular efficiency and/or at or above a particular price.
3. *Logistic curves are used to approximate distribution functions:* For mathematical simplicity, we will model product distributions with a logistic function, which for practical purposes is equivalent to a normal distribution, but which has a simpler mathematical form.
4. *Log-price and log-efficiency are the key quantities of interest:* We will assume that the log of appliance price and the log of appliance efficiency are quantities that are logistically distributed.

We will now review each of these modeling assumption in order.

MODELING DYNAMICS WITH LEARNING CURVES

There is an extremely large literature on learning curves dates back to the 1930's.

Sometimes the literature makes a distinction between learning curves and experience curves. The term learning curve is used when a change in cost can be directly tied to a production learning process. Experience curves are a more general empirical phenomenon where it is observed that many price trajectories follow a power law even if a direct connection to a technological learning process may not be clear.

For the sake of our empirical model development we treat learning and learning rates as a mathematically defined quantity. In our definition, we are concerned with price as a function of cumulative experience or knowledge. In this context, we define the learning rate as the elasticity of price with respect to cumulative experience. The concept of elasticity is a very general economic concept which describes how one economic quantity depends on another economic quantity. Specifically the elasticity of a quantity *y* with respect to another quantity *x* is the "percent change of *y* with respect to a percent change in *x*." More rigorously it is the derivative of log *y* with respect to log *x*:

$$Elasticity = \frac{\partial y/y}{\partial x/x} = \frac{\partial(\ln(y))}{\partial(\ln(x))} = \varepsilon$$

When we are calculating the price elasticity with respect to cumulative experience, the elasticity is the negative of the learning coefficient, i.e. $\varepsilon = -b$. In general the learning coefficient may be a function of several other variables, though in this study we confine ourselves to the case of constant learning coefficient.

If incremental acquisition knowledge is proportional to annual production, then we have the following two ordinary differential equations that define a learning curve:

$$\frac{d\ln(P)}{d\ln(X)} = -b \tag{1}$$

$$\frac{d\ln(X)}{dt} = \frac{Q}{X} \tag{2}$$

Where *P* is the price, *Q* is the sales of product per unit time and *X* is the cumulative experience in units of total product sold. Note that the simplest solution to equation (1) is the familiar learning curve relationship: $(P/P_0) = (X/X_0)^{-b}$, where *b* is an empirically determined parameter and P_0 and X_0 represent the price and cumulative experience at a reference time[7].

Note that this pair of ordinary differential equations can be written is a slightly more compact form if we use the log-transformed variables $\pi = ln(P)$, and $\zeta = ln(X)$:

$$\frac{d\pi}{d\zeta} = -b \tag{3}$$

$$\frac{d\zeta}{dt} = \frac{Q}{X} \tag{4}$$

EXPRESSING MARKET DISTRIBUTIONS AS CUMULATIVE DISTRIBUTION FUNCTIONS

A key reason for modeling price dynamics by modeling the cumulative price distribution functions is because cumulative distributions are computationally robust given noisy empirical data.

Figure 1 illustrates the evolution of the distribution of refrigeration appliance efficiency in the European market from 1991 to 2009. By showing the distributions in terms of a cumulative distribution (i.e. the fraction of products with efficiencies equal to or less than the efficiency on the horizontal axis), the motion of the market can be seen very clearly. Pre-1997, the cumulative distribution of efficiencies was moving rather slowly, and then post-1997 the cumulative distribution function has accelerated substantially.

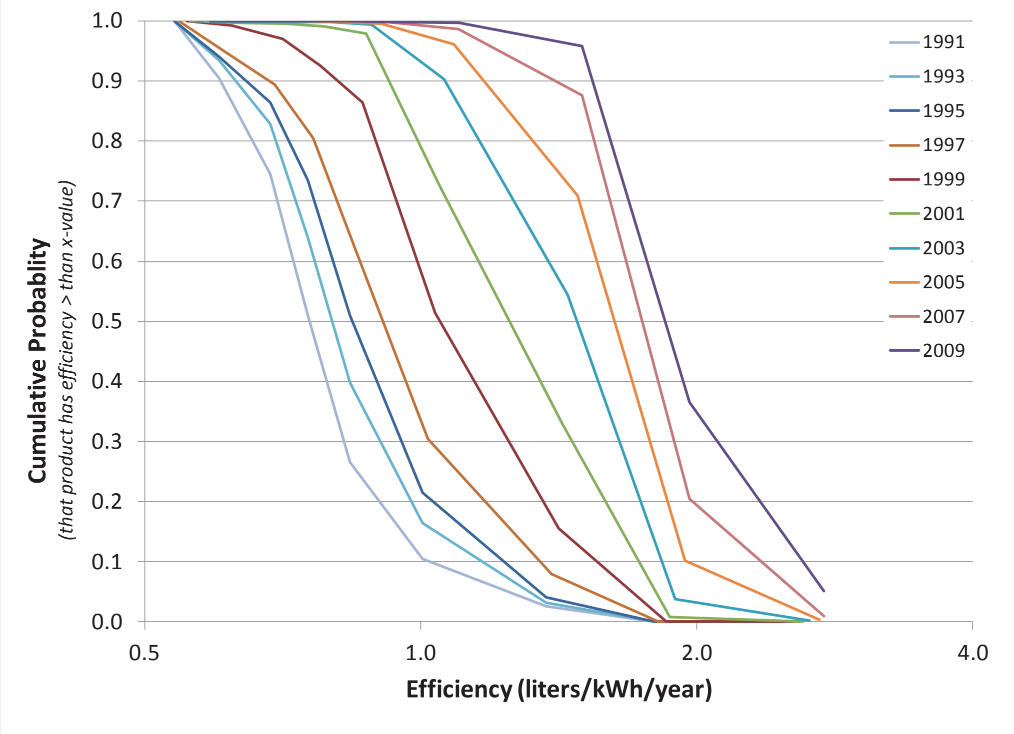

Figure 1. Evolution of the cumulative efficiency distribution of refrigeration products in the European market from 1991 to 2009. The average efficiency of the distribution increases as time progresses.

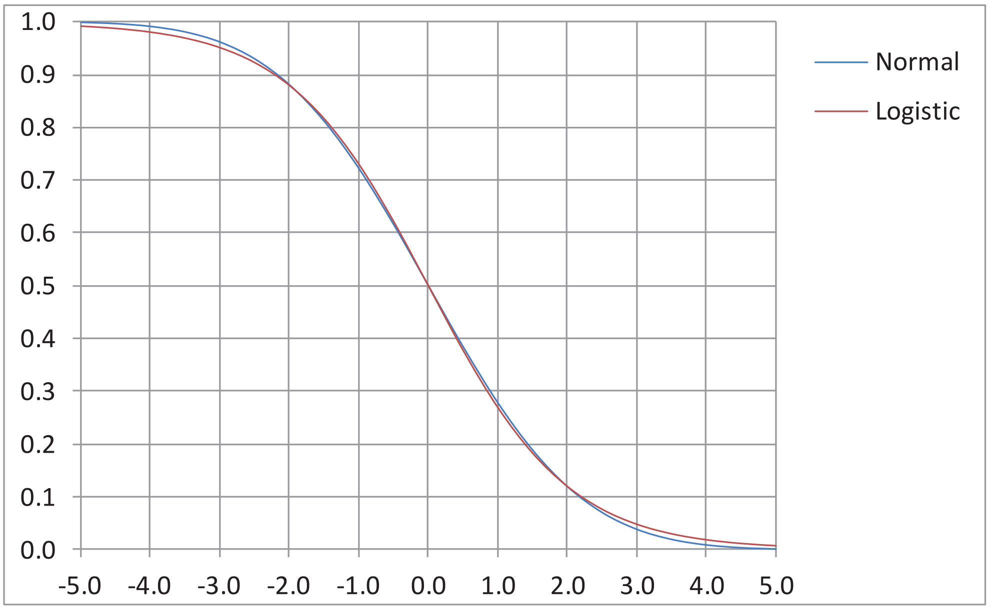

Figure 2. Comparison of logistic and normal cumulative frequency distributions, demonstrating that the two functions provide similar curves (with <0.01 RMS difference).

Note that the horizontal axis in Figure 1 is plotted on a logarithmic scale, meaning that rates of motion indicated in the plot are exponential.

USING LOGISTIC DISTRIBUTION FUNCTIONS

Because in most cases the data will not distinguish clearly between normally and logistically distributed data, we choose to use logistic distribution functions for analytic simplicity.

Figure 2 illustrates that very close similarity between logistic and normal probability distribution functions. The main difference between the two distributions is the tails of the distributions. A logistic distribution has an exponential tail, while a normal distribution has a tail that falls off as the exponential of the square of the distance from the mean.

The equations that describe the functional form of a cumulative logistic probability distribution are particularly simple.

The basic formula for a cumulative logistic distribution is:

$$F(x) = \frac{1}{1 + e^{\alpha(x - x_0)}} \tag{5}$$

where x_0 is the median of the distribution, and α determines the variance of the data about the mean.

Note that if we define the transformed variable ψ, where $\psi = \ln((1-F)/F)$, then the equation for the cumulative distribution simplifies to a simple linear equation:

$$\psi = \alpha(x - x_0) \tag{6}$$

where we have the inverse variable transformation: $F=1/(1+e^{\psi})$.

Figure 3 illustrates the cumulative distribution curves shown in Figure 1 in the transformed variable ψ. In the transformed variable, the S-curve shape of the cumulative distribution functions straightens to a set of curves that approximate a series of straight lines.

FORECASTING THE LOG OF PRICE AND THE LOG OF EFFICIENCY

Empirical data available on appliance price and energy use appears to indicate that price and energy use distributions may be rather highly skewed, and that they often evolve exponentially in time. We therefore develop our model equations in terms of log price and log efficiency.

Models developed for describing the dynamics and distributions of log price and log efficiency will be consistent with log-logistic price and efficiency distributions. In addition, exponential evolution of prices and efficiency will translate into a linear evolution of log price and log efficiency over time.

Figure 4 illustrates one example of the observation of a log-logistic distribution in appliance energy use measurements. This figure provides the cumulative distribution function for annual energy use measurements for a field survey of refrigeration energy use in Ghana in 2006.[13] A simple two-parameter log-logistic function fits the distribution with a root mean square (RMS) deviation of approximately one percent.

Observations of the EU Refrigerator Market

Correlation between price and market adoption: Figure 5 illustrates the distribution of market price as a function of the fraction of the market at or above an efficiency level. For the European refrigerator data, there appears to be a pre-2003 distribution, and a post-2003 distribution that are quite different.

We note that 2003 is the year in which the A+ and A++ levels were created by European Commission Directive 2003/66/EC.[14]

Correlation between prices and cumulative shipments: Figure 6 illustrates the declining prices of EU refrigerators in terms of the cumulative shipments of refrigerators at or above each of the different efficiency levels. The figure shows how declines in the cost of efficient refrigerators accelerate once cumulative shipments exceed 1 to 10 million units. Refrigeration appliances in the A+ and A++ categories are initially defined in 2003 and introduced into the market after the policies on standards and labels were implemented. Upon market entry, the A+ and A++ levels initially have average prices lower than the A and B level appliances had at market introduction. This is consistent with the observation of distinct pre-2003 and post-2003 price distributions in Figure 5.

Formulation of Price-Efficiency Model

We now integrate our observations of price-efficiency dynamics in the European refrigerator market into a mathematical model that can be used to quantitatively describe our observa-

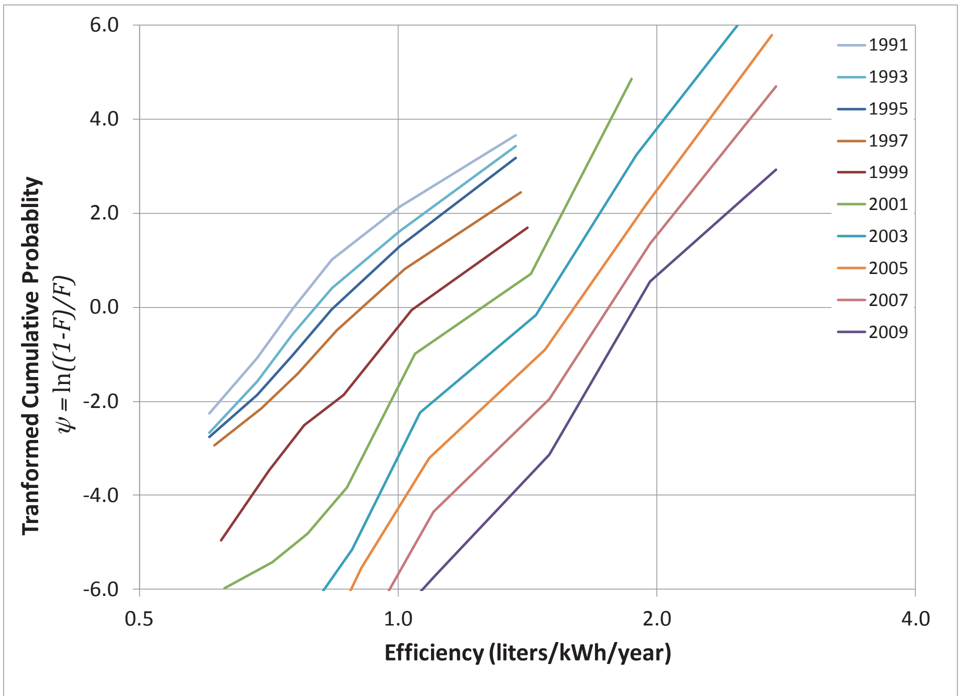

Figure 3. Cumulative distribution curves illustrated in Figure 1 plotted in the transformed variable ψ = ln((1-F)/F). Note that the coordinate transformation converts the "S-curves" of the cumulative distribution functions into curves that approximate straight lines.

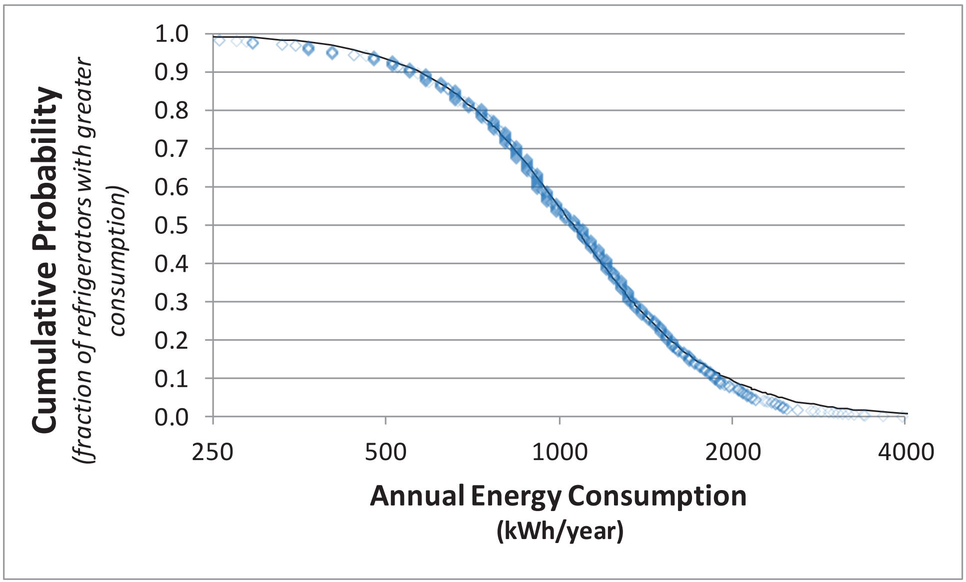

Figure 4. Cumulative distribution function of refrigeration appliance energy use in Ghana. The symbols are the data points, and the black curve is a log-logistic distribution fit to the data points.

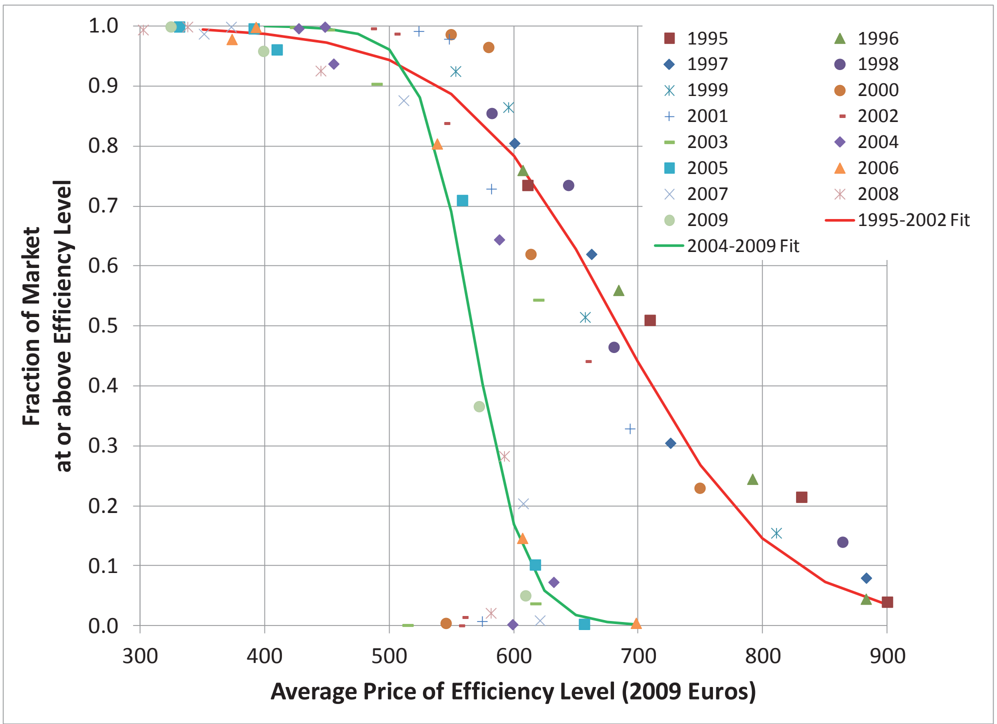

Figure 5. The fraction of the market at or above an efficiency level as a function of the average refrigerator price at that efficiency level. The two curves are cumulative logistic functions fit to pre-2003 and post-2003 data.

tions, and potentially generalize them to other appliances and markets.

MODELLING FRAMEWORK

Figure 7 illustrates the mathematical framework that we use for modeling price-efficiency dynamics. In this framework there is an initial condition that specifies the price-efficiency distribution in the start year, a bottom of the market (BOM) boundary condition that specifies the price of products as they become obsolete and leave the market, and a top of the market (TOM) boundary condition that specifies the price and efficiency of new products entering the market.

From the initial condition and the boundary conditions, the model calculates the dynamics of the central price-efficiency distribution over time.

Specifically, we structure our model with the following inputs and assumptions:

A. *Initial price-efficiency distribution:* We solve the price-efficiency dynamics as an initial value problem. At some time,

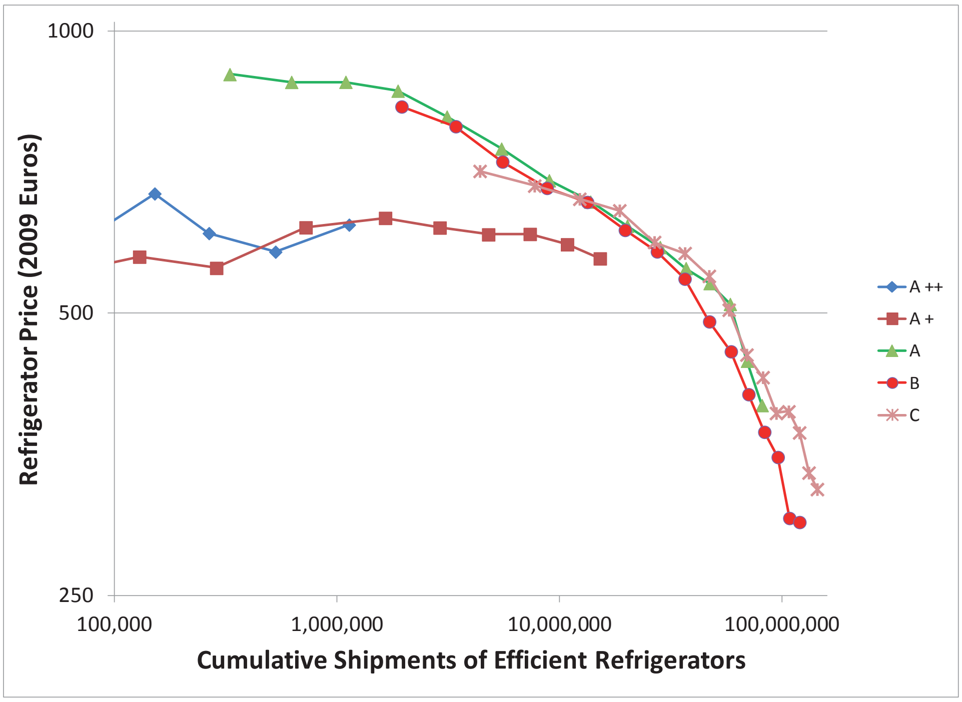

Figure 6. Price as a function cumulative shipments for different efficiency levels of EU refrigeration appliances.

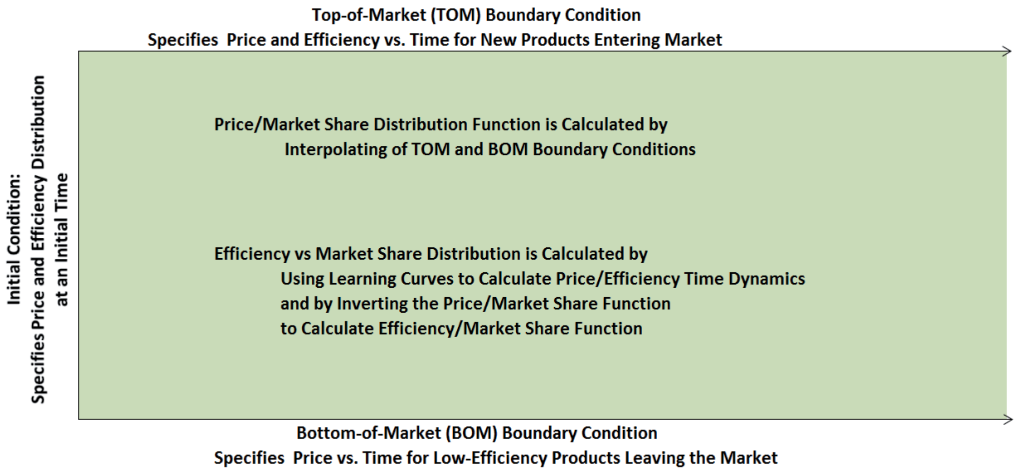

Figure 7. Framework for modeling price-efficiency dynamics. The light green shaded area represents the computational domain of the mathematical model. Given an initial condition, and boundary conditions at the top of the market and bottom of the market, the model equations estimate the dynamics of the price-efficiency distribution in the middle of the market. Because the model is assuming that technological learning is a key driver for price changes, initial prices and efficiencies need to be specified for new products entering at the top of the market.

there is a known price-efficiency distribution, and over time the price-efficiency distribution evolves based on market conditions.

B. *Top-of-market boundary condition:* We define a top-of-market boundary condition that specifies the price and efficiency at market introduction as a function of time; Top-of-market is defined as a particular level in the cumulative distribution function (e.g. 1 % market share);

C. *Bottom-of-market boundary condition:* We define a bottom-of market condition which is willingness to pay (or price) at the bottom of the market, but the efficiency is not specified; Bottom-of-market is defined as a particular level in the cumulative distribution function (e.g. 99 % market share);

D. *Price distribution:* The cumulative market distribution for price is assumed to be a logistic interpolation of the top-of-market and bottom-of-market conditions;

E. *Price vs. efficiency:* The price at a particular efficiency level follows the learning curve equations (3) and (4) specified above;

F. *Top-of-market initial cumulative experience:* At the top-of-market boundary, in addition to price, an initial cumulative experience (representing initial knowledge generated by research and development prior to market introduction) is specified for initiating the integration of equation (4);

G. *Determination of efficiency distribution:* The market distribution of efficiencies is determined by matching the price as a function of efficiency (calculated from the learning curve equations) with the price distribution interpolated from the top-of-market and bottom-of-market boundary conditions:

H. *Model inputs:* The input data needed for the model includes the initial efficiency distribution, the top-of-market price-efficiency function, and the bottom-of-market price.

I. *Fitted model parameters:* The fitted model parameters that are not directly observable in market data include the learning rate and potentially the initial cumulative learning at efficient product market introduction.

With these model inputs and assumptions, we now formulate the details of the model equations, and the computational steps necessary to make a price-efficiency forecast.

The modeling steps include:

1. Specifying market data in transformed product market variables.
2. Specifying the initial price-efficiency distribution.
3. Specifying TOM price-efficiency and BOM price.
4. Calculating the interpolated price-efficiency distribution.
5. Determining the initial value and TOM boundary condition for the experience variable.
6. Solving the dynamic equations for efficiency and price.

In the following subsections, we describe each of these steps in detail and illustrate them with data from the European refrigeration appliances market.

USING TRANSFORMED PRODUCT MARKET VARIABLES

The first step in modeling market price-efficiency dynamics is rescaling to the market variables in which the market distributions and dynamics take their simplest form.

Our market data and variables before transformation are quantities such a cumulative market share, *F*, price, *P*, efficiency, *Eff*, and cumulative experience, *X*. The transformations of these variables that we use are:

$\psi = \ln((1-F)/F)$	(transformed market share)
$\pi = \ln(P)$	(log price)
$\phi = \ln(Eff)$	(log efficiency)
$\zeta = \ln(X)$	(log experience)

With these transformed variables, the equations describing the price-efficiency model take on a particularly simple form, as we will show in the following sections.

INITIAL CONDITION FOR THE PRICE-EFFICIENCY DISTRIBUTION

If the efficiency and price distributions in the market data fit logistic distributions as discussed earlier in this paper, then specifying the initial condition for the price-efficiency distribution is particularly simple because both log price, π, and log efficiency, ϕ, are linear in the transformed market share variable, ψ, as follows:

$$\pi(\psi, t_0) = \pi_0 + \alpha \cdot \psi \tag{7}$$

$$\phi(\psi, t_0) = \phi_0 + \beta \cdot \psi \tag{8}$$

where t_0 is the initial time, π_0 is the median price at the initial time, ϕ_0 is the median efficiency at the initial time, and α and β are the appropriate coefficients for the initial logistic distribution fits to the price and efficiency distributions. Note that if we use these equations to solve price as a function of efficiency, we get the following equation:

$$(\pi - \pi_0) = \frac{\alpha}{\beta}(\phi - \phi_0) \tag{9}$$

which implies a familiar power law relationship between price and efficiency

$$P = P_0 \left(\frac{Eff}{Eff_0} \right)^{\frac{\alpha}{\beta}} \tag{10}$$

For the European refrigeration appliances case, if we chose t_0 = 1995, then the parameters for the initial condition distributions are as follows: π_0 = 6.55, α = 0.0911, ϕ_0 = -0.152, and β = 0.144. Hence, the initial price-efficiency power law exponent is approximately 0.63.

PRICE-EFFICIENCY AT TOM BOUNDARY AND PRICE AT BOM BOUNDARY

In our modeling framework, the dynamics of the price-efficiency distribution depends on TOM and BOM boundary conditions. The TOM and BOM boundary conditions depend in part on the resolution with which one wants to perform the market modeling. Theoretically products exist in a market if there is at least one product sold within a finite amount of time, but more practically, we set a particular market share level: 0.1 %, 1 %, 2 %, 5 %, etc. that defines when a product type has entered or left the market.

Given the resolution of the data that we have for the European refrigerated appliances market, we set the market entry point as 0.25 % market share, and the market exit point as a cumulative market share of 99.75 % of products with efficiencies equal to or greater than the particular price or efficiency level. These values correspond to a transformed market share variable of approximately $-6 < \psi < 6$.

To characterize the behavior observed in the European refrigeration appliance market, we specify the following TOM and BOM market conditions:

- For efficiency at TOM, we estimate an average improvement rate of 3 % per year.
- For price at TOM, price we estimate 1,325 Euro at 1995, remaining constant until 1998, and then decreasing to 800 Euro between 2000 and 2004, and remaining constant at 800 Euro after 2004.

- For price at BOM, it remains constant at approximately 370 Euro for the entire period.

Given these TOM and BOM boundary condition specifications, we can calculate the model dynamics. If there are deviations between model results and the observed data, we can adjust the details of the specifications to improve the statistical performance of the model fit with the data.

Note that we estimated the detailed parameters of the TOM and BOM boundary conditions by fitting the model results to the available market data, as described in the model results section below.

INTERPOLATING THE PRICE DISTRIBUTION FUNCTION

In our transformed variables the interpolation of the price distribution function is particularly simple. The interpolation of the price between the TOM and BOM boundary condition is linear in ψ, which means that the log price is given by the equation:

$$\pi(\psi,t)=\frac{(\pi_{TOM}(t)-\pi_{BOM}(t))}{(\psi_{TOM}-\psi_{BOM})}\cdot\psi \tag{11}$$

where ψ_{TOM} is the top-of-market value of the transformed market share variable (ψ = 6 in our case), ψ_{BOM} is the bottom-of-market value of the transformed market share variable (ψ = -6 in our case), $\pi_{TOM}(t)$ is the top-of-market value of the log price which can vary with time, and $\pi_{BOM}(t)$ is the bottom-of-market value of the log price that can vary over time but which we set to a constant value as described above.

INITIAL VALUES AND TOM BOUNDARY CONDITION FOR THE EXPERIENCE VARIABLE

We note that there is one additional set of boundary conditions that is needed for solving equations (12) and (13). Specifically, we need to set the initial value and the top-of-market boundary condition for the cumulative experience variable:

$$\zeta(\phi,t)=\ln(X(\phi,t))$$

We estimate the initial condition of $\zeta(t_0)$ with available data by extrapolating the shipments and the efficiency distribution back in time. Specifically we examine shipments at and above each efficiency level for the first five years of the available sales data. For each of these time series, we also calculate the relative (exponential) rate of increase, γ, of shipments in the first five years of data (1995–1999 inclusive). Then assuming an exponential back-extrapolation of the data, we estimate the cumulative experience as: $X(t_0) = Q(t_0)/\gamma$. Where for the data we have for the European refrigeration appliances case we select t_0 = 1995.

The results of this calculation are presented in Figure 8, which expresses in the cumulative experience in millions of units, and then plots the log of cumulative experience as a function of the transformed market share variable ψ.

The equations provided in Figure 8 provides an initial TOM boundary value for ζ of -3.96 at ψ=6. But we notice that the top of market price value changes with time as specified above. In specifying the TOM boundary condition for ζ, we assume that the ζ boundary condition changes in a way that is consistent with the learning equations (i.e. equation (3) above). In particular, we specify that the following TOM boundary condition holds:

$$\zeta_{TOM}(t)=\zeta_{TOM}(t_0)-\frac{\pi_{TOM}(t)-\pi_{TOM}(t_0)}{b} \tag{12}$$

SOLVING THE EQUATIONS FOR PRICE-EFFICIENCY DYNAMICS

In this section we describe how the dynamics of the price-efficiency relationship is calculated in the model using learning curve equations.

First we note that equations (3) and (4) described previously can be rewritten in the following form:

$$\frac{\partial\pi(\phi,t)}{\partial t}=-b\frac{\partial\zeta(\phi,t)}{\partial t} \tag{13}$$

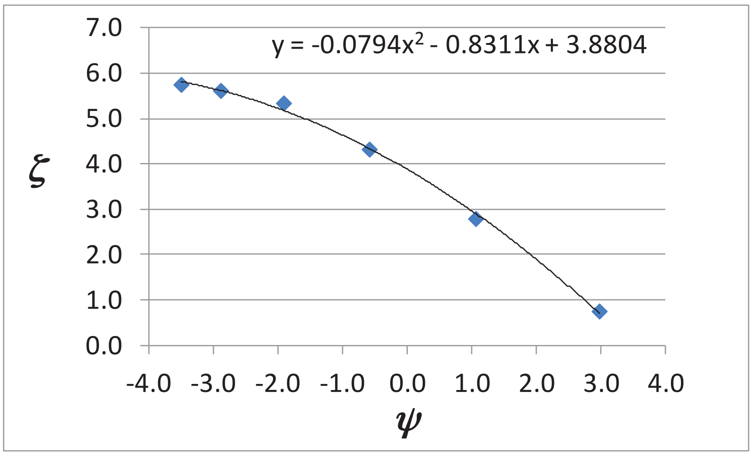

*Figure 8. Log cumulative experience as a function of the transformed market share variable at the initial time, 1995 for European refrigeration appliances. Cumulative experiences is calculated in units of millions of shipments. This means that a value of ζ = 4 corresponds to a cumulative experience of e^4*1000000 = 54,598,150 cumulative shipments.*

$$\frac{\partial \zeta(\phi,t)}{\partial t} = \frac{Q_0(t)}{(1+e^{\psi(\phi,t)})} \frac{1}{X(\phi,t)} \quad (14)$$

Equation (13) is the technological learning equation which describes the relative rate of price decline at a particular efficiency which is proportional to the rate at which cumulative experience is increasing at that efficiency. Equation (14) is the formula that describes the relative rate at which cumulative experience grows at a particular efficiency. The first term is the fraction of product sales that are at or above a particular efficiency level, where $Q_0(t)$ is the total product sales at a particular time, and $1/(1+e^{\psi(\phi,t)})$ is the market share factor at the particular efficiency and time.

When we are solving equations (13) and (14), we would prefer to calculate price, efficiency and experience as a function of time, *t,* and the market share variable, ψ. To transform equation (12) into a form where ψ is the market coordinate in which we are solving the equation, we note the following relationship between partial derivatives:

$$\frac{\partial}{\partial t}[\pi(\phi(\psi,t),t)] = \frac{\partial \pi}{\partial \phi}\frac{\partial \phi(\psi,t)}{\partial t} + \frac{\partial \pi(\phi,t)}{\partial t} \quad (15)$$

and solving for the derivative of efficiency with respect to time at constant market share variable, we get:

$$\frac{\partial \phi(\psi,t)}{\partial t} = \frac{\frac{\partial \pi(\psi,t)}{\partial t} - \frac{\partial \pi(\phi,t)}{\partial t}}{\frac{\partial \pi(\phi,t)}{\partial \phi}} \quad (16)$$

which provides the equation that gives us the derivative of log-efficiency with respect to time at constant market share variable as the ratio of the difference of log-price derivatives (at constant market share and constant efficiency), and the derivative of the log-price with respect to log-efficiency. To solve equation (16), we calculate $\delta\pi(\psi, t)/\delta t$ by taking the partial derivative of equation (11) with respect to time. We calculate $\delta\pi(\phi, t)/\delta t$ by using equation (13), and we calculate $\delta\pi(\phi, t)/\delta\phi$ by taking a numerical derivative of log price with respect to log efficiency at the current time step. This then allows us to use equation (16) to numerically solve for log efficiency, $\phi(\psi, t)$, as a function of market share variable and time.

Similarly, we wish to calculate the cumulative experience variable, ζ, as a function of the market share variable, ψ, and time, *t*. We therefore write an equation similar to equation (15) for ζ:

$$\frac{\partial \zeta(\psi,t)}{\partial t} = \frac{\partial \zeta}{\partial \phi}\frac{\partial \phi}{\partial t} + \frac{\partial \zeta(\phi,t)}{\partial t} \quad (17)$$

To solve equation (17), we calculate $\delta\zeta(\phi, t)/\delta t$ by using equation (14), and we calculate $\delta\zeta(\phi, t)/\delta\phi$ by taking a numerical derivative at the current time step, and we use the solution of equation (16) to provide an estimate of $\delta\phi(\psi, t)/\delta t$.

Model Results

When we numerically solve the model equations and fit them to the European refrigerator data, we find that there are five parameters that need to be adjusted to provide an optimal fit to the data. These include (with the optimum fitted values in parentheses):

- Technological learning rate (*b*=1.3).
- Top-of-market efficiency improvement rate (3 %/year).
- Bottom-of-market price (370 Euros).
- Top-of-market price pre-1998 (1,325 Euro).
- Top-of-market price post-2004 (800 Euro).

Figures 9 and 10 illustrate the performance of the model in describing the observations of efficiency distribution dynamics and price-efficiency dynamics for the European refrigeration appliances market. Note that the 1995 conditions are specified as initial conditions, but the performance of the model in the later years illustrates that it may possible to model the vast majority of market price-efficiency dynamics using a technological learning framework.

Conclusion

In this study, we have developed a novel set of model equations that can be used to describe the evolution of appliance efficiency distributions and price-efficiency correlations over time. A key advantage of this model is its empirical foundation and parametric simplicity. The model depends only on four key parameters and specification of a set of initial conditions. The four key parameters are a learning rate (or cumulative experience price elasticity), a top-of-market price, a bottom-of-market price, and a top of market efficiency improvement rate that corresponds to the top-of-market price level that is selected.

We fit our model to data from the European refrigerated appliances market and found that the corresponding best fit parameter values are: a learning exponent of *b*=1.3, a top-of-market efficiency improvement rate of 3 %/year, a top-of-market (at 0.25 % market share) price of 800 Euro post-2004 and 1,325 Euro pre-1998, and a bottom of market price of 370 Euro.

Future research is likely to explore whether the model equations presented here can be used to reliably forecast the future evolution of efficiency distributions and price-efficiency correlations in product markets. In addition we expect future research to examine the connections between model parameters and public policies including technology research investments, and the impacts of standards and labeling programs on market prices and dynamics.

Endnotes

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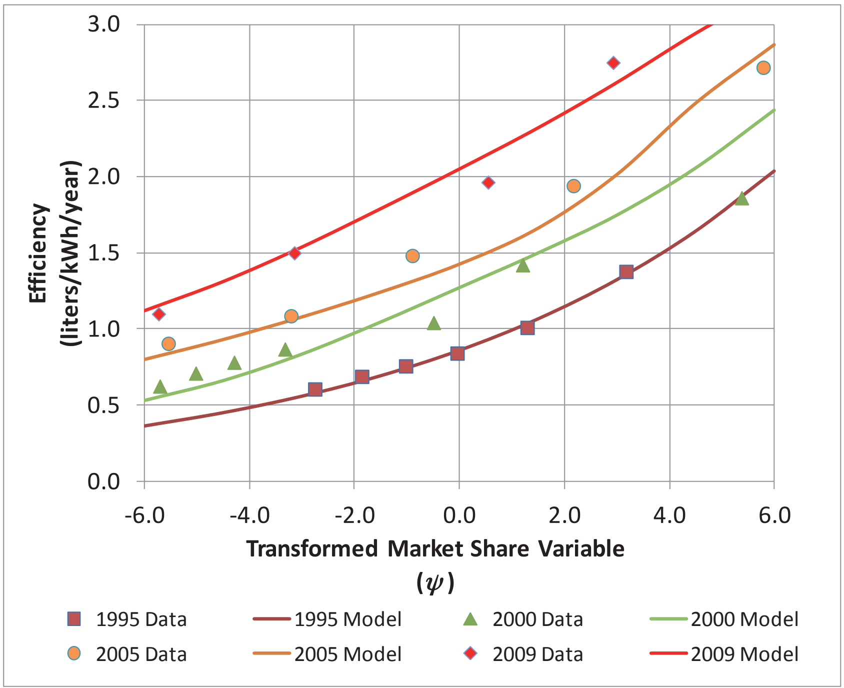

Figure 9. Comparison of model results and data for the efficiency vs. market share relationship. Note that the relationship in 1995 is set as an initial value of the market dynamics in the model equations, so the model and the data fit very closely. Also note that there are three model parameters: three boundary condition parameters, and a technological learning rate that were adjusted to fit the model to the data.

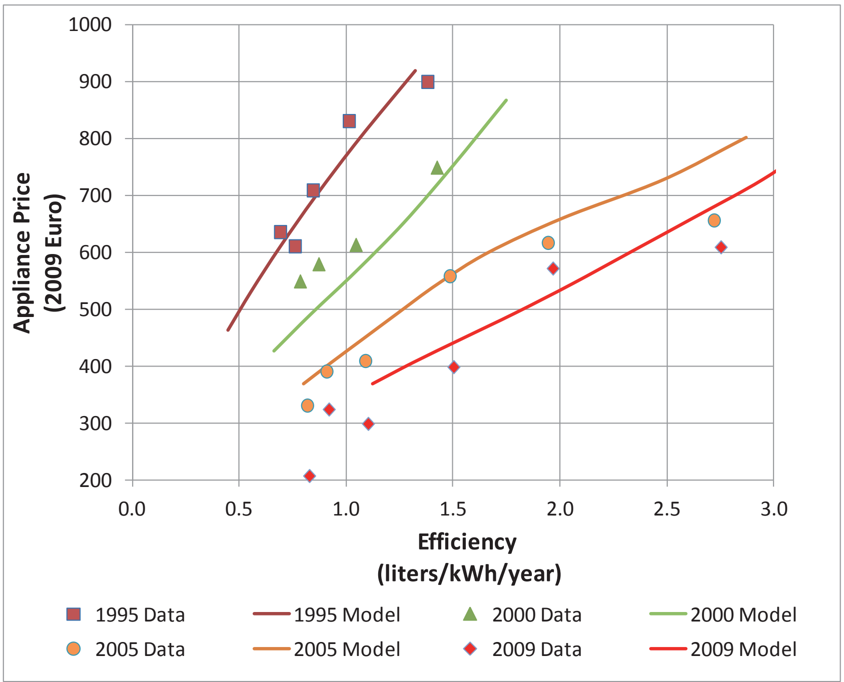

Figure 10. Comparison of model results and data for the price vs. efficiency relationship. Note that the relationship in 1995 is set as an initial value of the market dynamics in the model equations, so the model and the data fit very closely in 1995. Also note that there are three model parameters: three boundary condition parameters, and a technological learning rate that were adjusted to fit the model to the data.

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14 See http://eur-lex.europa.eu/LexUriServ/LexUriServ.do?uri=OJ:L:2003:170:0010:0014:EN:PDF.