Modelling the Dynamics of Appliance Price-Efficiency Distributions

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NOTE: Updated 3 July: conversion problems from ppt to pdf fixed

Motivation: Solving the Policy Problem

For efficient appliance markets, policy is optimized when marginal costs equal marginal benefits:

Marginal Cost $\Delta P_A = -\Delta PVOC = -\Delta UEC \cdot \overline{P_E} \cdot \sum_{n=1}^{L} \frac{1}{(1+i)^n}$

Marginal Benefit

We can solve this where: $PWF = \sum_{n=1}^{L} \frac{1}{(1+i)^n}$ $\quad Eff = \frac{1}{UEC}$

and $elasticity = \varepsilon = \frac{\Delta P_A / P_A}{\Delta Eff / Eff} = \frac{\partial \ln(P_A)}{\partial \ln(Eff)}$

Solving the Policy Optimization Problem

Simplified Solution: $$Eff(t) = \frac{\overline{P_E}(t) \cdot PWF}{P_A(t) \cdot \varepsilon(t)}$$

To optimize the efficiency trend targeted by policy, the policy analyst needs to know:
How will prices and the elasticity of purchase price with respect to efficiency change over time?

In other words…

We need to monitor or forecast the Price-Efficiency relationship to optimize the efficiency trend for the appliance policy in question

Why We Need Partial Differential Equations (PDEs) for the Forecast

Different Levels of Mathematical Complexity solve Different Problems:

1. Algebra provides equations that we can fit to data that relate one quantity to another
2. Ordinary Differential Equations (ODEs) solve problems where the rate of change of a quantity is equal to an algebraic equation
3. Partial Differential Equations (PDEs) solve problems where rates of change with respect to multiple variables are related to equations that depend on multiple variables

We have multiple rates of change and multiple variables in the Price-Efficiency forecast problem, which requires the use of PDEs:

- **Derivatives of price are functions of both efficiency and time**
- **Technology learning says price depends on cumulative sales (yet another variable), and sales depend on price**

Let the Mathematical Gymnastics Begin!

Math Trick #1: ***The experience curve is just a solution to an elasticity equation***

$$Elasticity = \frac{\Delta y / y}{\Delta x / x} = \frac{\partial(\ln(y))}{\partial(\ln(x))} = -b$$

Experience Curve Equation: $P = X^{-b}, \ln(P) = -b\ln(X)$

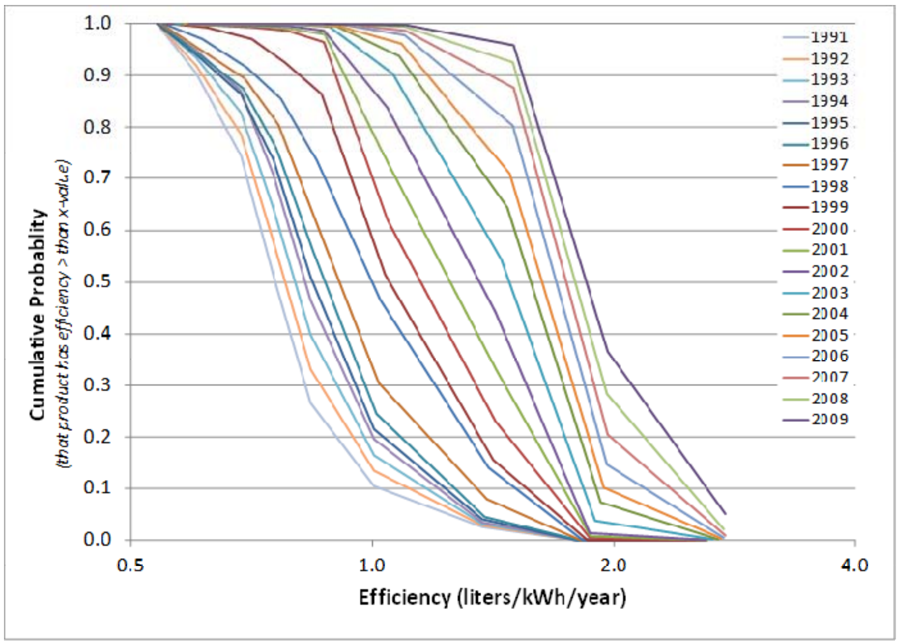
$$\frac{d\ln(P)}{d\ln(X)} = -b \qquad \frac{d\ln(X)}{dt} = \frac{Q}{X}$$

And if… $\pi = \ln(P)$, $\zeta = \ln(X)$ **Then…**

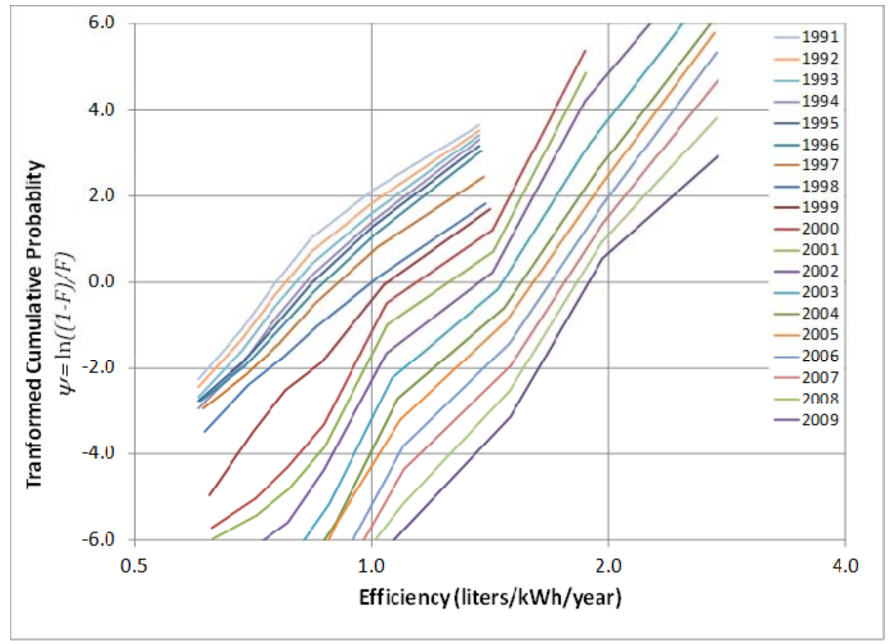
$$\frac{d\pi}{d\zeta} = -b \qquad \frac{d\zeta}{dt} = \frac{Q}{X}$$

More Math Gymnastics: This one is difficult!

<u>Math Trick #2</u>: *Write cumulative market distribution functions in terms of a transformed market position variable*



Before transformation



After transformation

$$F(Eff) = \frac{1}{1+e^{\alpha(Eff - Eff_0)}} \qquad \psi = \ln((1-F)/F) \qquad F = 1/(1+e^{\psi}) \qquad \psi = \alpha(Eff - Eff_0)$$

More Math Gymnastics: These equations are non-intuitive!

<u>Math Trick #3</u>: *Price is a function of cumulative shipments at or above the efficiency level*

$$\frac{\partial \pi(\phi,t)}{\partial t} = -b\frac{\partial \zeta(\phi,t)}{\partial t}$$

$$\frac{\partial \zeta(\phi,t)}{\partial t} = \frac{Q_0(t)}{(1+e^{\psi(\phi,t)})}\frac{1}{X(\phi,t)}$$

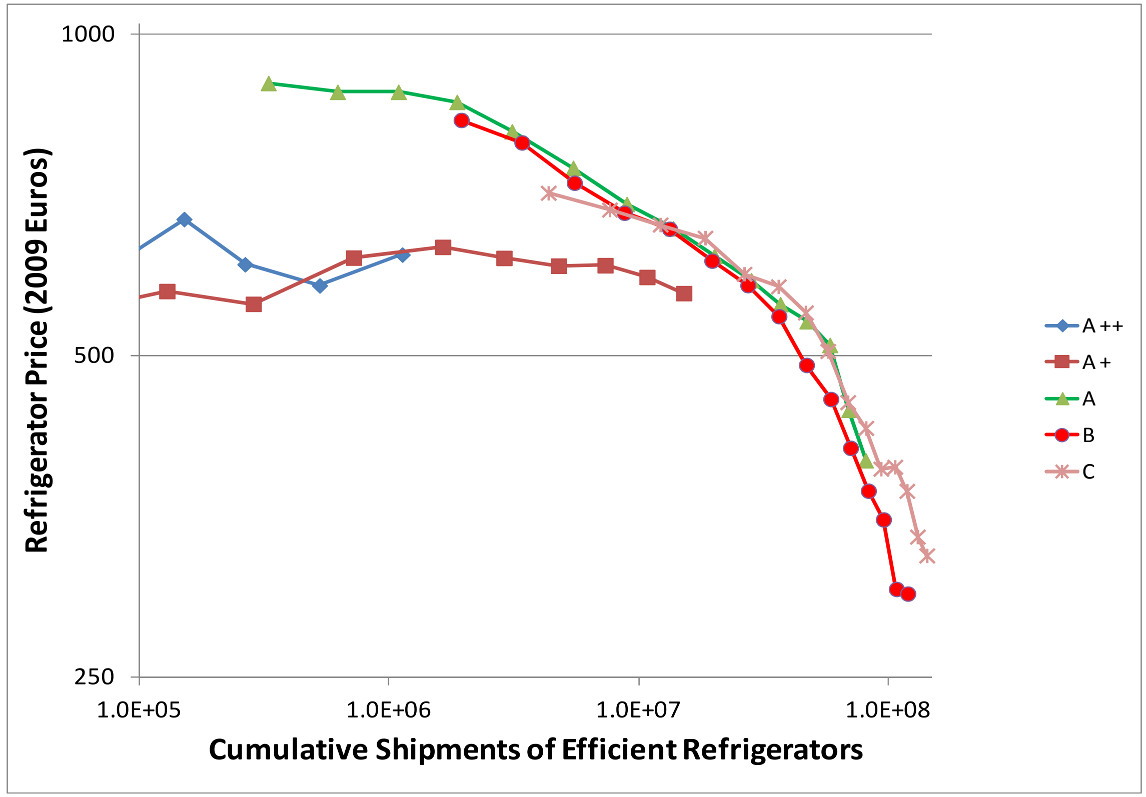


Figure 6: *Price as a function cumulative shipments for different efficiency levels of EU refrigeration appliances.*

Modeling Framework

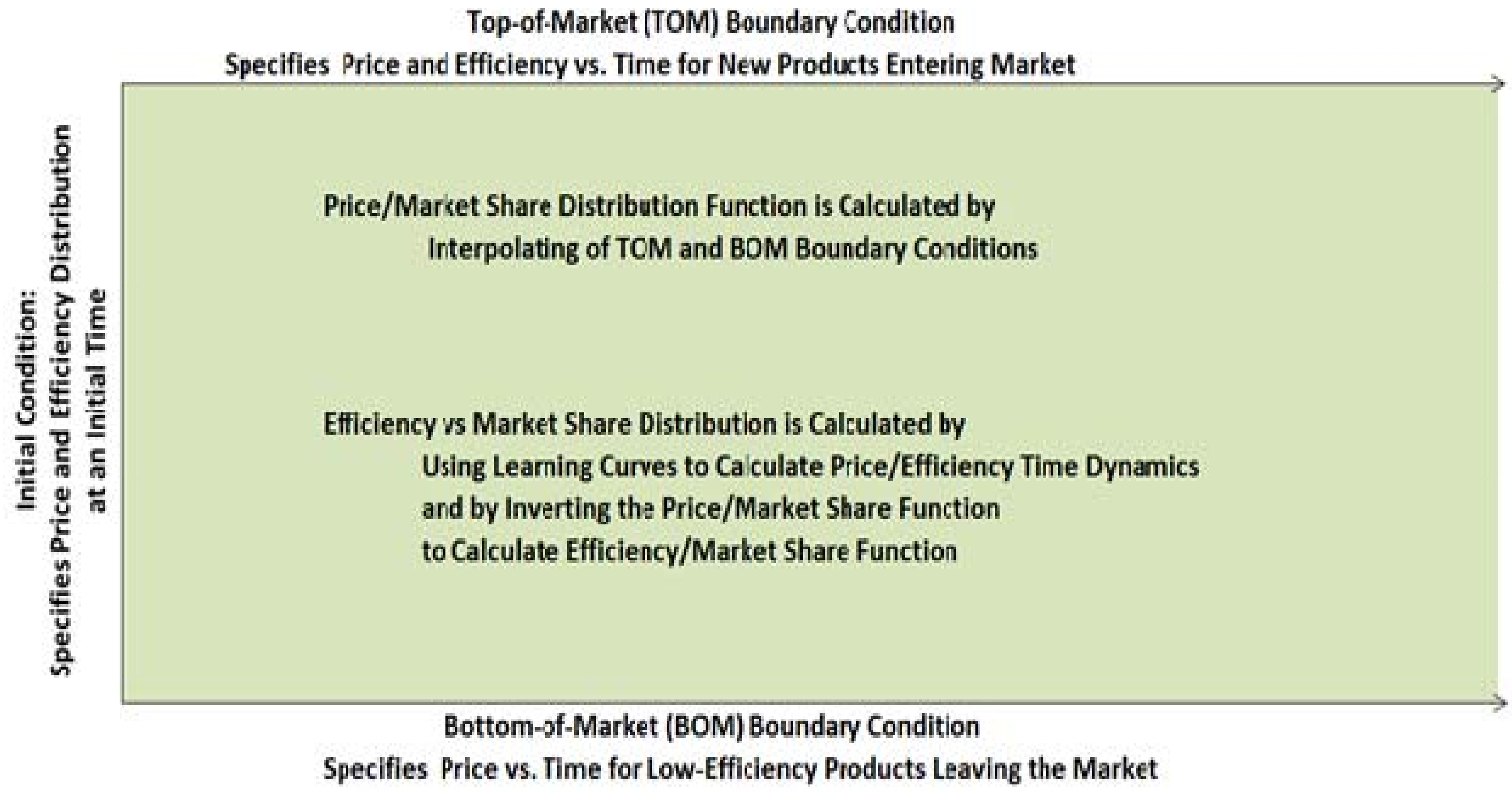


Figure 7: *Framework for modeling price-efficiency dynamics. The light green shaded area represents the computational domain of the mathematical model. Given an initial condition, and boundary conditions at the top of the market and bottom of the market, the model equations estimate the dynamics of the price-efficiency distribution in the middle of the market. Because the model is assuming that technological learning is a key driver for price changes, initial prices, efficiencies, and cumulative experience values need to be specified for new products entering at the top of the market.*

Modeling Steps

1. Specify market data in transformed product market variables
2. Specify the initial price-efficiency distribution
3. Specify top-of market (TOM) price-efficiency and bottom-of-market (BOM) price
4. Calculate the interpolated price-efficiency distribution
5. Determine the initial value and TOM boundary condition for the experience variable
6. Solve the dynamic equations for efficiency and price

Model Variables

$$\psi = \ln((1-F)/F)$$ **(transformed market share)**

$$\pi = \ln(P)$$ **(log price)**

$$\phi = \ln(Eff)$$ **(log efficiency)**

$$\zeta = \ln(X)$$ **(log experience)**

Initial Condition for Price-Efficiency

$$\pi(\psi, t_0) = \pi_0 + \alpha \cdot \psi$$

$$\phi(\psi, t_0) = \phi_0 + \beta \cdot \psi$$

$$(\pi - \pi_0) = \frac{\alpha}{\beta}(\phi - \phi_0)$$

$$P = P_0 \left(\frac{Eff}{Eff_0} \right)^{\frac{\alpha}{\beta}}$$

For the European refrigeration appliances case, if we chose t_0 = 1995, then the parameters for the initial condition distributions are as follows: π_0 = 6.55, α = 0.0911, ϕ_0 =-0.152, and β =0.144. Hence, the initial price-efficiency elasticity is approximately 0.63.

Price Distributions

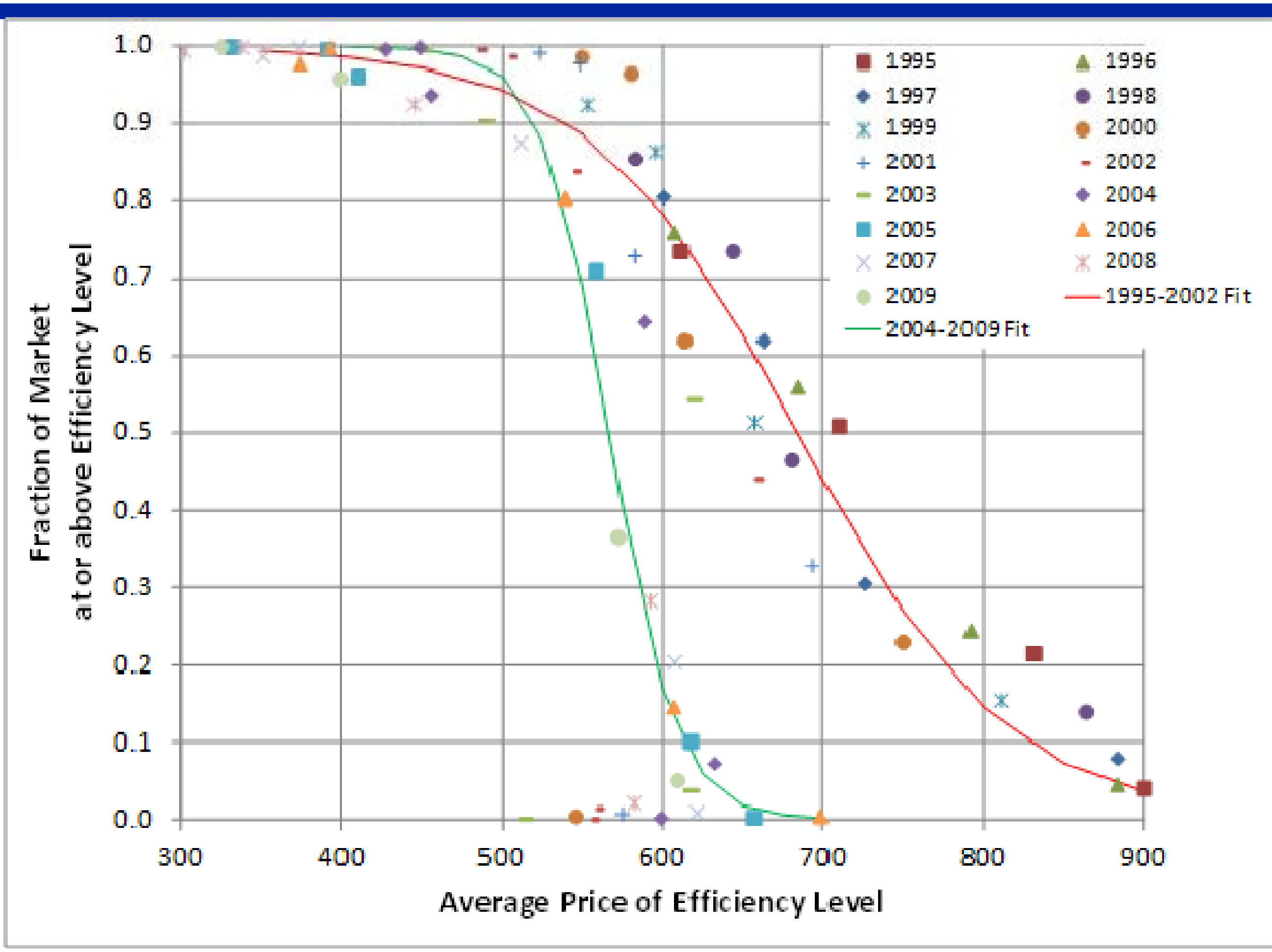


Figure 5: ***The fraction of the market at or above an efficiency level as a function of the average refrigerator price at that efficiency level. The two curves are cumulative logistic functions fit to pre-2003 and post-2003 data***

TOM & BOM Boundaries

- **For efficiency at TOM (ψ = 6), we estimate an average improvement rate of 3% per year.**
- **For price at TOM, price we estimate 1325 Euro at 1995, remaining constant until 1998, and then decreasing to 800 Euro between 2000 and 2004, and remaining constant at 800 Euro after 2004.**
- **For price at BOM (ψ=-6), it remains constant at approximately 370 Euro for the entire period.**

$$\pi(\psi,t)=\frac{(\pi_{TOM}(t)-\pi_{BOM}(t))}{(\psi_{TOM}-\psi_{BOM})}\cdot\psi$$

Initial Condition for Log Cumulative Experience

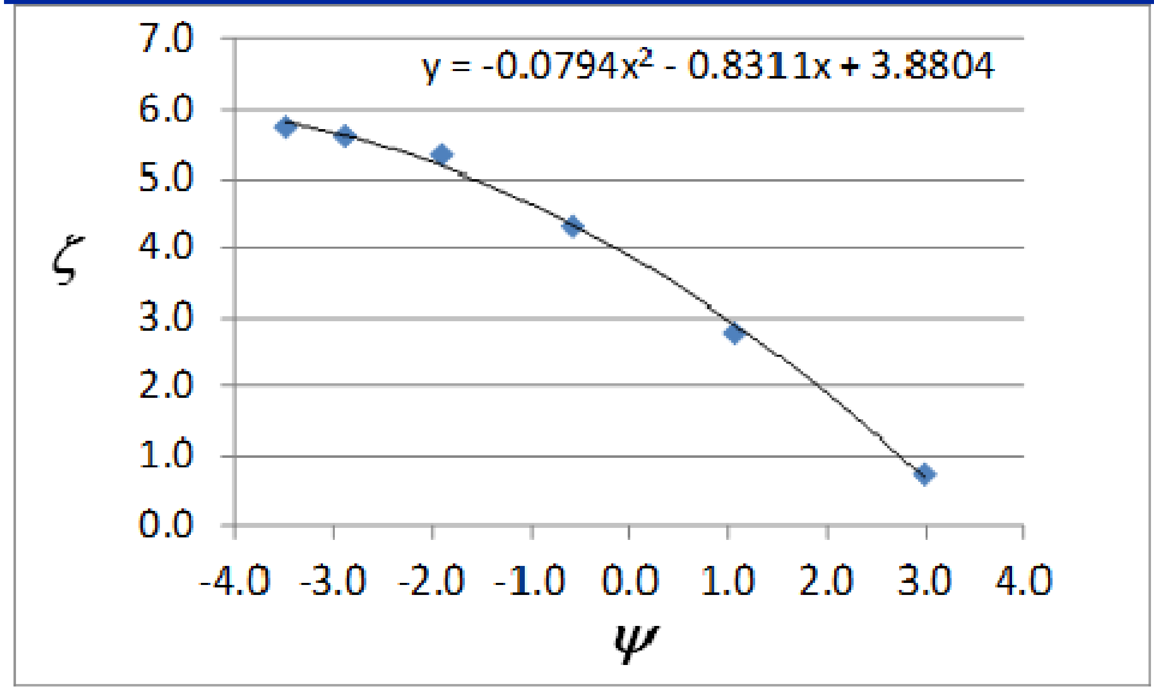


Figure 8: ***Log cumulative experience as a function of the transformed market share variable at the initial time, 1995 for European refrigeration appliances. Cumulative experiences is calculated in units of millions of shipments. This means that a value of z = 4 corresponds to a cumulative experience of e^4*1000000 = 54,598,150 cumulative shipments.***

$$\zeta_{TOM}(t) = \zeta_{TOM}(t_0) - \frac{\pi_{TOM}(t) - \pi_{TOM}(t_0)}{b}$$

Solving the Dynamic Equations

First calculate change in experience as a function of efficiency:

$$\frac{\partial \zeta(\phi,t)}{\partial t} = \frac{Q_0(t)}{(1+e^{\psi(\phi,t)})} \frac{1}{X(\phi,t)}$$

Then calculate change in price as a function of efficiency:

$$\frac{\partial \pi(\phi,t)}{\partial t} = -b \frac{\partial \zeta(\phi,t)}{\partial t}$$

Update price as a function of market position:

$$\frac{\partial}{\partial t}[\pi(\phi(\psi,t),t)] = \frac{\partial \pi}{\partial \phi} \frac{\partial \phi(\psi,t)}{\partial t} + \frac{\partial \pi(\phi,t)}{\partial t}$$

Solving the Dynamic Equations

And update efficiency as a function of market position:

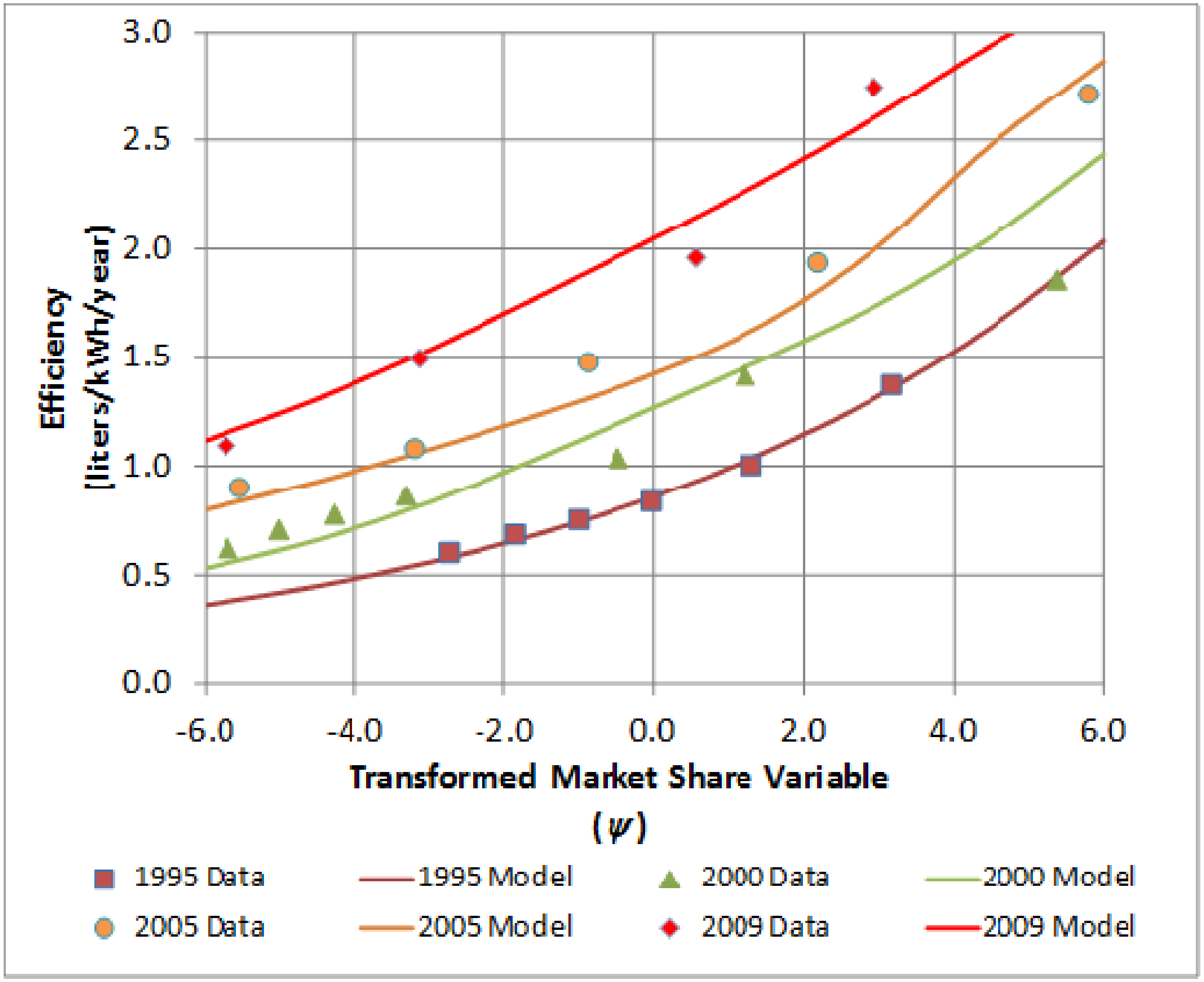
$$\frac{\partial \phi(\psi,t)}{\partial t} = \frac{\dfrac{\partial \pi(\psi,t)}{\partial t} - \dfrac{\partial \pi(\phi,t)}{\partial t}}{\dfrac{\partial \pi(\phi,t)}{\partial \phi}}$$

Update cumulative experience as a function of market position:

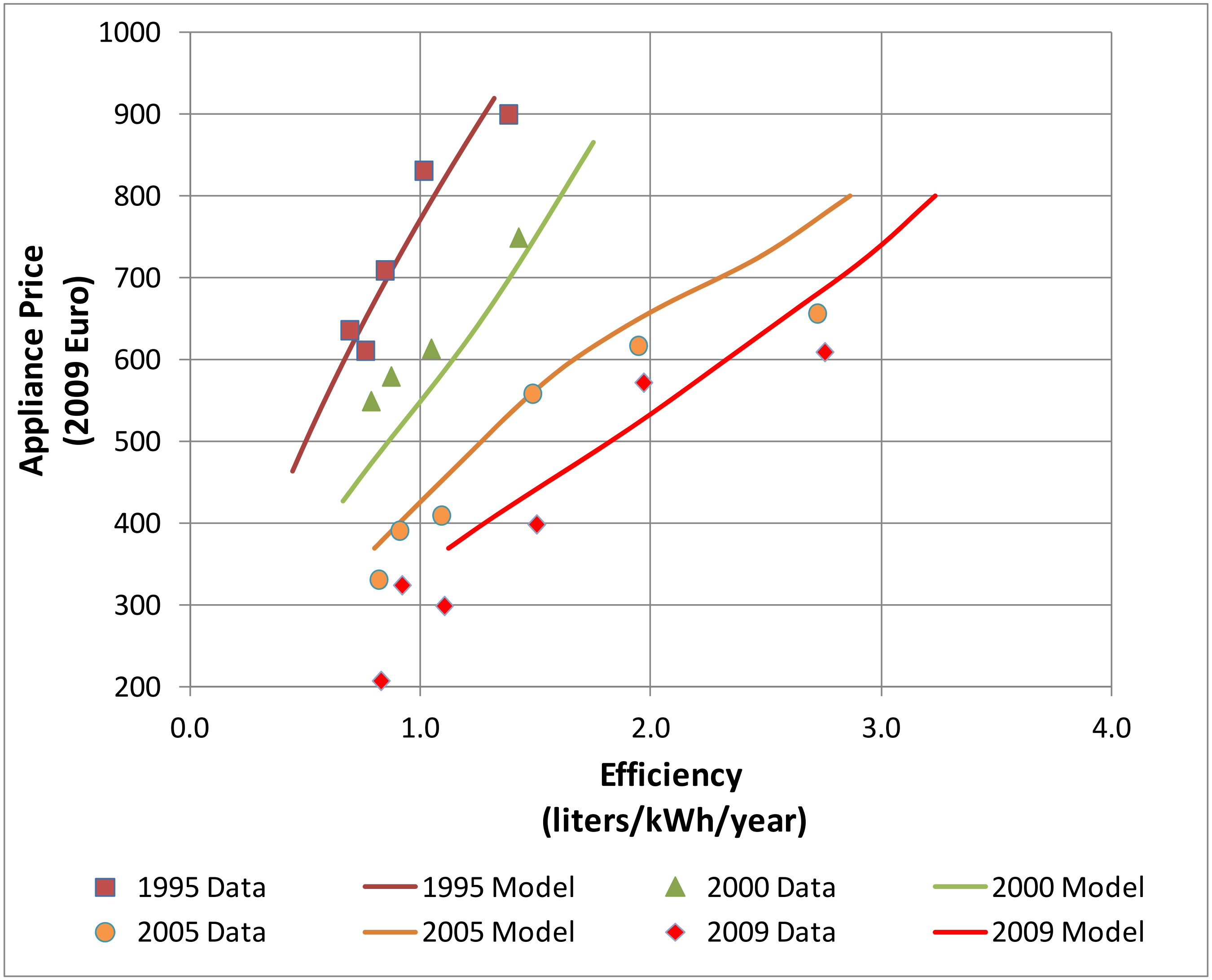
$$\frac{\partial \zeta(\psi,t)}{\partial t} = \frac{\partial \zeta}{\partial \phi}\frac{\partial \phi}{\partial t} + \frac{\partial \zeta(\phi,t)}{\partial t}$$

Result: Price and experience are updated as a function of efficiency, and price, experience and efficiency are updated as a function of market position at each time step.

Model Results: Efficiency Distributions



Model Results:
Price-Efficiency Functions



Model Results:
LCC Hindcast and Forecast