Carbon tax and substitution effects in the French industrial sector: an econometric assessment

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Abstract
Within the political framework of the "Grenelle de l'environnement" in France, the French government is studying various fiscal measures to encourage actors to reduce CO_2 emissions, among others a carbon tax on every fossil energy source. The efficiency of such a measure is directly linked to the price responsiveness of the actors concerned. In this paper, after a survey of the different possible forms for an energy demand function, we focus on the secondary sector of the French economy (after having removed the industrial sub-sectors concerned with double usage or non-energy use of fuels) and assess the likelihood of industrialists shifting from one energy source to another due to a change in the relative prices of different energy sources (coal, heavy fuel oil, heating oil, natural gas and electricity), besides the improvements in energy efficiency.

We conclude that with price variations of the magnitude that was observed between 1986 and 2004 the substitution effects remain low: industrialists were much more likely to improve the energy efficiency of their appliances and processes than to shift energy sources in response to a given increase in prices. Significant substitution effects, for example after applying a carbon tax, would probably only occur for greater price variations. However, the actors' response (interfuel substitution) to an increase in the price of coal is 5 to 10 times higher than for other energy sources. The study also gives us information about the speed at which industrialists adapt to variations in prices, and the results have already been used for the assessment of future fiscal measures in France.

Introduction
Carbon taxes are one of the major means of providing an incentive to reduce greenhouse gas emissions, along with tradable emission certificates or regulation; within the political framework of the "Grenelle de l'environnement" in France, it has become important to evaluate the optimal level of a possible carbon tax in France and its effects on the economy.

The level of the tax is set according to forecasts about its economic effect (implying an effect on reducing emissions: environmental effectiveness), so the latter is the main concern. Apart from the income effect leading to a reduction in consumption and improvements in energy efficiency, a carbon tax would lead to a substitution effect between the different energy sources. This effect is essential for assessing the efficiency of a carbon tax, since the carbon content can be very different depending on the energy source (fossil fuels and electricity). Thus the level of carbon tax per energy unit varies significantly between energy sources, leading to a possible substitution effect between energy sources which, in turn, leads to changes in the total CO_2 emissions of the economy.

In this paper, we deal with the econometric evaluation of the substitution effects between energy sources in the secondary sector of the French economy, which implies choosing a model for estimating energy demand and evaluating its parameters. The industrial sub-sectors concerned with double usage or non-energy use of fuels were removed from the aggregated secondary sector that was studied for the econometric evaluation.

This work has been used for a first estimation of the effects of a carbon tax on the economy.

We will first present a survey of the different possible forms for an energy demand function, linking our presentation whenever possible to the existing computable general equilibrium (CGE) models. Next we present the demand function we built for our econometric estimation; finally we present our empirical results and conclude.

Energy demand functions used currently: an overview

GENERALITIES ON DEMAND FUNCTIONS

The demand function generally derives from a production or cost function. Given a certain output demand Y^d, the demand function is the result of the maximization of profit, or symmetrically the minimization of a cost function, typically:

$$C = \sum_{i=1}^{n} P_i X_i \tag{1}$$

under the constraint of the production function:

$$F(X_1 ... X_n) = Y^d \tag{2}$$

where n is the number of production factors, P_i is the price of production factor i and X_i the amount of factor i used. Regarding the question of energy substitution, each of the n factors is typically a given energy source.

As we can see, the central question is the form of the production function. Several well-known functional forms were first introduced for the two-factors case: for example, the Cobb-Douglas function initially introduced by Wicksell followed by Cobb and Douglas in 1928 or the CES function introduced by Arrow, Chenery, Minhas and Solow (Arrow, Chenery, Minhas and Solow 1961). Their extension to the n-factors case is not entirely obvious; it mainly depends on the key definition of the elasticity of substitution. While some functional forms impose very strict limitations on this elasticity of substitution, other forms are much more flexible, and are indeed called flexible functional forms.

We will first give an overview of the different possible definitions for the elasticity of substitution. Then we will examine the different production functions used in the literature, distinguishing them according to their properties, especially regarding the elasticity of substitution.

THE ELASTICITY OF SUBSTITUTION

The definition of the elasticity of substitution in the n-factors case is an attempt to generalize the definition in the 2-factors case. In the case of only two different goods, the elasticity of substitution σ_{12} between two production factors 1 and 2 was originally defined by Hicks in 1932, and can be written (Frondel 2004):

$$\sigma_{12} = \frac{1}{S_2}\eta_{12}, \text{ where } \eta_{12} = \frac{\partial \ln X_1}{\partial \ln P_2} \text{ and } S_2 = \frac{X_2 P_2}{YP} \tag{3}$$

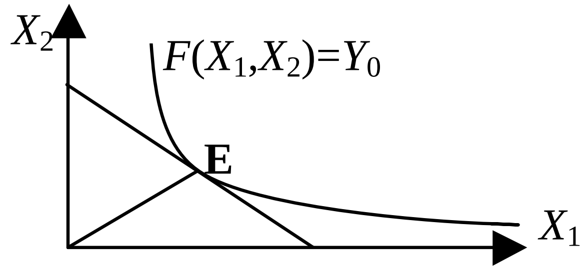

Figure 1. The elasticity of substitution is the measure of the curvature of the isoquant at point E.

where η_{12} is by definition the cross-price elasticity between factors 1 and 2, S_2 is the share of the total revenue imputable to factor 2, X_i is the level of input of factor i, P_i the price of factor i, Y the total revenue and P the output price.

The aim of the elasticity of substitution is to provide "a measure of the ease with which the varying factor can be substituted for others" (Hicks 1932). Its main difference compared with cross-price elasticity is the fact that cross-price elasticity is computed in a virtual situation where all the other terms remain constant. In fact, the formal definition of the elasticity of substitution corresponds to the measure of the curvature of the isoquant defined by:

$$F(X_1, X_2) = Y_0 \tag{4}$$

where Y_0 is a given level of output (see Figure 1).

The generalization to the n-factors case is not self-evident; several different definitions have been proposed. The one perhaps most commonly used in the literature is the Allen Elasticity of Substitution. Allen and Hicks proposed in 1934 the following definition for the elasticity of substitution between factors i and j (Frondel 2004):

$$AES_{ij} = \frac{\partial \ln(X_i / X_j)}{\partial \ln(P_j / P_i)} = \frac{1}{S_j}\eta_{ij}, \text{ where } \eta_{ij} = \frac{\partial \ln X_i}{\partial \ln P_j}$$

$$\text{and } S_j = \frac{X_j P_j}{YP} \tag{5}$$

where η_{ij} is by definition the cross-price elasticity, S_j is the share of total revenue imputable to factor j, and X_i, Y and P are defined the same way as for the 2-factors case. With this definition, the Allen Elasticity of Substitution appears to be a formally straightforward generalization of the 2-factors Hicks Elasticity of Substitution.

The AES is by far the most used form of elasticity of substitution in the literature (Frondel 2004); yet Blackorby and Russell (Blackorby and Russell 1989) pointed out using examples that the AES was not a good measure of the ease of substitution and adds no more information to that contained in the cross-price elasticity η_{ij}. In particular, it "provides no information about relative factor shares (the purpose for which the elasticity of substitution was originally defined [by opposition to the cross-price elasticity])" (Blackorby and Russell 1989). Other measures of the ease of substitution have since emerged, among them the Morishima elasticity of substitution, first introduced by Morishima in 1967 (Blackorby and Russell 1981):

$$MES_{ij} = \frac{\partial \ln(x_i / x_j)}{\partial \ln P_j} \quad (6)$$

This definition corresponds to the Allen elasticity of substitution in the case where a change in P_j/P_i is solely due to a change in P_j (Frondel 2004). It is the only one that does not impose symmetry between factors *i* and *j* (that is, it does not impose $\sigma_{ii} = \sigma_{ij}$), which seems to be more intuitive in the *n*-factors case (Blackorby and Russell 1989). It gives a more direct vision of the percentage change in relative shares induced by a given percentage change in a relative price since it also has the following property (Blackorby and Russell 1989):

$$\frac{\partial \ln\left(P_i X_i / P_j X_j\right)}{\partial \ln(P_i / P_j)} = 1 - MES_{ij} \quad (7)$$

GENERAL CONSTRAINTS ON PRODUCTION FUNCTIONS

Before presenting the different existing production, cost and demand functions, we present the mathematical constraints that are generally required for every production function. They are generally based on well-established economic principles, and are listed for example in Uzawa (1962) or Blackorby and Russell (1981).

The first condition is quasi-concavity (or often strict quasi-concavity). This means by definition that the isoquants of the considered function are convex, as for example in Figure 1 for the 2-factors case. This ensures that there is an optimal distribution of factors; if the function is strictly quasi-concave, this solution to the optimization problem is unique. Formally, quasi-concavity corresponds to:

$$C(Y) = \left\{X = (X_1,...,X_n) / F(X) = Y\right\} \text{is convex in } \Re^n \quad (8)$$

The second condition is homogeneity of degree one: this simply means that a homothetic increase in all the levels of inputs should result in an equivalent increase in output, and is synonymous with constant returns to scale. While this condition is not always verified for a small number of firms, it is consistent with intuition for a large number of firms at the national level. Formally, the homogeneity condition corresponds to:

$$\forall k > 0, F(kX_1,...,kX_n) = k.F(X_1,...,X_n) \quad (9)$$

The last condition is the presence of partial derivatives of any order. This is a continuity condition that allows much greater simplicity in the mathematical treatment of the functions.

CONSTANT ELASTICITY OF SUBSTITUTION (CES) PRODUCTION FUNCTIONS

We can now introduce the first great family of production functions, by far the most used till now (Kemfert 1998). In the attempt to generalize the production functions of the 2-factors case, this family of functional forms was inferred from properties (7), (8) and (9), from the derivability condition and from a hypothesis of constancy of the elasticities of substitution, that is, by analogy with the 2-factors case, that the elasticities of substitution do not depend on the actual cost shares of the different production factors:

$$\forall (S_1,...,S_n) \in [0,1]^n,\ \sigma_{ij}(S_1,...,S_n) = \sigma_{ij}^0 \in \Re \quad (10)$$

Note that this simply means, that the elasticities of substitution remain the same whatever the situation is, in terms of cost share for each production factor. It does not *a priori* imply that the elasticities of substitution are all *equal,* that is, for each i and j, $\sigma_{ij} = \sigma_0$ with σ_0 a given constant.

We have not yet stated which definition of the *n*-factors elasticity of substitution we were using. In fact, this theoretical approach has been followed for each possible definition of the elasticity of substitution; it can be found in the founding articles of Uzawa (Uzawa 1962) for the Allen elasticity of substitution and Kuga (Kuga 1979) followed by Blackorby and Russell (Blackorby and Russell 1989) for the Morishima elasticity of substitution; McFadden (McFadden 1963) did the same work for other forms of elasticity of substitution not presented here, with similar results. These different authors came to the remarkable conclusion that the hypothesis of constancy of the elasticity of substitution implied an almost unique form of production function, the *n*-factors CES production function, with possible nestings, with very close results whatever definition of the elasticity of substitution is chosen.

The *n*-factors CES production function

The *n*-factors CES production function for the k[th] firm of the economy is of the form:

$$F(X_{k1},...,X_{kn}) = \left(\sum_{i=1}^{n} \alpha_i X_{ki}^{\frac{\eta-1}{\eta}}\right)^{\frac{\eta}{\eta-1}} \text{ with } \eta > 0,$$

$$\alpha_i > 0 \text{ and } \sum_{i=1}^{n} \alpha_i = 1 \quad (11)$$

Except in one particular case (CES production functions nested in a Cobb-Douglas production function, with the Allen elasticity of substitution being equal to 1 between different nested production functions, see Uzawa 1962), this is the most general production function that can be expected using the general constraints presented above and the hypothesis of constant elasticities of substitution. These hypotheses also lead to the striking supplementary consequence of equality of elasticities of substitution:

$$\forall i, j, \quad \sigma_{ij} = \eta \in \Re \quad (12)$$

All the elasticities of substitution are equal to the parameter η in Equation (11) (see below for the demonstration). This shows that the CES hypothesis is very restrictive; in fact intuitively the elasticity of substitution between two given factors should vary somewhat according to their respective market shares.

The limit cases where $\eta \to 0$ or $\eta \to 1$ lead respectively to the *n*-factors Leontief (13) or Cobb-Douglas (14) production functions (with the same constraints on the α_i):

$$F(X_{k1},...,X_{kn}) = \min_{1<i<n}\left(\alpha_i X_{ki}\right) \quad (13)$$

$$F(X_{k1},...,X_{kn}) = \prod_{i=1}^{n} X_{ki}^{\alpha_i} \quad (14)$$

These production functions are simply the limit cases of the general form (11) with a null elasticity of substitution (no substitution effects are possible, the factors are complementary and their relative proportions are determined once and for all) or with a unit elasticity of substitution respectively.

Deriving a demand function in the CES case

We now demonstrate how a demand function can be derived from the CES production function, taking our inspiration from Dixit and Stiglitz (1977), Van der Mensbrugghe (2005) and Reynès (2006). As was stated above, given a certain output demand Y_k^d, the demand function is the result of the minimization of the following cost function:

$$C_k = \sum_{i=1}^{n} P_i X_{ki} \quad (15)$$

under the constraint of the production function given in Equation (11):

$$F(X_{k1}...X_{kn}) = Y_k^d \quad (16)$$

The Lagrangian function for this optimization problem is:

$$\Lambda = C_k - \lambda\left(F(X_{k1}...X_{kn}) - Y^d{}_k\right) \quad (17)$$

The first-order conditions are:

$$\Lambda'(X_{ki}) = 0,\ \Lambda'(\lambda) = 0 \quad (18)$$

The second-order conditions that guarantee that the optimum is a maximum are verified due to the fact that the cost function and the production function are quasi-concave (see above). The first-order conditions imply:

$$\forall i_0,\quad P_{i_0} = \lambda \left(\sum_{i=1}^{n} \alpha_i X_{ki}^{\frac{\eta-1}{\eta}} \right)^{\frac{1}{\eta-1}} \alpha_{i_0} X_{ki_0}^{\frac{-1}{\eta}} \quad (19)$$

which leads to:

$$\frac{X_{ki}}{X_{ki_0}} = \frac{\alpha_{i_0}}{\alpha_i}\left(\frac{P_i}{P_{i_0}}\right)^{-\eta} \quad (20)$$

and then, if we reformulate the cost function (15) for a given i_0:

$$P_{i_0} X_{ki_0} + \sum_{\substack{i=1 \\ i\neq i_0}}^{n} P_i X_{ki} = C_k \quad (21)$$

and insert Equation (20):

$$X_{ki_0}\left(P_{ki_0} + \sum_{\substack{i=1 \\ i\neq i_0}}^{n} \frac{\alpha_{i_0}}{\alpha_i}\left(\frac{P_i}{P_{i_0}}\right)^{-\eta} \right) = C_k \quad (22)$$

we finally get the following form for the demand function of the k^{th} firm for factor i:

$$X_{ki_0} = \frac{C_k}{mP}\left(\frac{P_{i_0}}{P}\right)^{-\eta} \quad (23)$$

with:

$$P = \left(\frac{1}{m}\sum_{i=1}^{n} P_i^{1-\eta}\right)^{\frac{1}{1-\eta}} \quad (24)$$

where m is the number of firms in the economy and P is called the CES dual price (Dixit and Stiglitz 1977).

Finally, the aggregated demand function for factor i_0 for the whole economy can be calculated by summing the Equations (23) for each k:

$$X_{i_0} = X^m\left(\frac{P_{i_0}}{P}\right)^{-\eta} \quad (25)$$

with:

$$X^m = \frac{\left(\frac{1}{m}\sum_{k=1}^{m} C_k\right)}{P} \quad (26)$$

We can also calculate the Allen and Morishima elasticities of substitution, starting from definitions (5) and (6) and Equation (20):

$$AES_{ij} = \frac{\partial \ln(X_i / X_j)}{\partial \ln(P_j / P_i)} = \frac{\partial \ln\left(\frac{\alpha_j}{\alpha_i}\left(\frac{P_i}{P_j}\right)^{-\eta}\right)}{\partial \ln(P_j / P_i)} = \eta;$$

$$MES_{ij} = \frac{\partial \ln\left(\frac{\alpha_j}{\alpha_i}\left(\frac{P_i}{P_j}\right)^{-\eta}\right)}{\partial \ln p_j} = \eta \quad (27)$$

In conclusion, in the case of the CES production function, the Allen and Morishima elasticities of substitution are both equal to η.

An extension: N-level nested CES production functions

The N-level nested CES production functions (see for example Rutherford 1995) are an extension of the CES production function, consisting of a hierarchical structure of CES production functions nested in each other. For example, for a 2-levels nested CES production function, the set $\{X_1...X_n\}$ of factors is subdivided into n_0 subsets $\{X_{11}...X_{1n_{11}}\}$ to $\{X_{n_01}...X_{n_0 n_{1n_0}}\}$ of length n_{1j}, and the production function can be written:

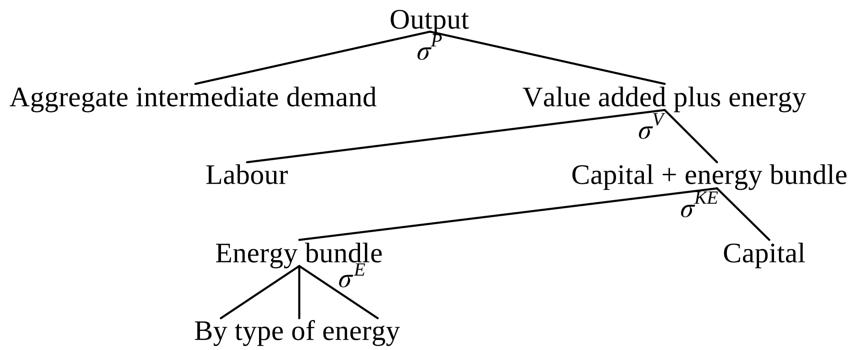

Figure 2. An example of a 4-levels nested CES production function with the different elasticities of substitution, inspired by the ENV-Linkage model (Van der Mensbrugghe 2005)

$$F_0 = \left(\sum_{j=1}^{n_0} \alpha_{0j} F_{1j}^{\frac{\eta_0 - 1}{\eta_0}} \right)^{\frac{\eta_0}{\eta_0 - 1}} \quad \text{with } \forall j, \qquad (28)$$

$$F_{1j} = \left(\sum_{i=1}^{n_{1j}} \alpha_{1ji} X_{ji}^{\frac{\eta_{1j} - 1}{\eta_{1j}}} \right)^{\frac{\eta_{1j}}{\eta_{1j} - 1}} \quad \text{with } \eta_0 > 0, \quad \alpha_{0j} > 0,$$

$$\sum_{j=1}^{n} \alpha_{0j} = 1, \text{ and } \forall k, i, \quad \eta_{ki} > 0, \quad \alpha_{ki} > 0$$

$$\text{and } \sum_{i=1}^{n_{1k}} \alpha_{ki} = 1$$

These production functions are very well suited when a production factor can be disaggregated between several other underlying production factors: for example, for the decomposition of energy between the different sources of energy (see Figure 2). However, in this example the elasticity of substitution between each energy source remains constant and equal across all the energy sources. Since there is no natural hierarchy between energy sources as, for example, between energy sources on one side and capital and labour on the other, this form of production function leads to the same relationships between energy sources (in terms of elasticities of substitution) as a simple CES production function.

Conclusion on the CES: uses

In conclusion, the CES production function and its special cases (Cobb-Douglas or Leontief) are widely used in CGE models (Kemfert 1998), often in the form of a nested CES production function. For example, and in the particular field of models assessing the impact of climate taxes, the ENV-Linkage model (Van der Mensbrugghe 2005, OECD 2008) uses precisely such nested CES production functions, where all energy sources are grouped into a single energy bundle. The Gemini-E3 model (Bernard and Vielle 2000) describes the electricity generation sector using a very detailed CES production function: it first makes a distinction between the different energy sources, then between fossil energy sources and the others, then for each energy source, between capital and energy.

But even in the latter case, the substitution between each fossil energy source is modelled by a single CES function. The main problem of the CES production functions (and all their derivatives) remains the equality between the elasticities of substitution for each combination of energy sources: for each i and j, $\sigma_{ij} = \sigma_s$, where σ_s is the elasticity of substitution for the CES production function encompassing the energy sources. This would mean that the ease of substitution from one fossil energy source to every other would be the same.

FLEXIBLE FUNCTIONAL FORMS

Presentation of the concept

The flexible functional forms are the other great family of production, cost and demand functions. Unlike the CES functions, they do not derive from the hypothesis of constancy of elasticity of substitution, which has proven to be highly restrictive, and in particular far too restrictive for sophisticated technologies (see also Christensen, Jorgenson and Lau 1972, Guilkey, Lovell and Sickles 1983). Once the CES hypothesis is removed, many functional forms for a production function are possible, the only constraint being the general constraints presented above. Many different forms have been developed in the last 40 years; some of those most commonly used in the literature are generalized Leontief (Diewert 1971), translog (Christensen, Jorgenson and Lau 1973), and linear logit (Considine and Mount 1984).

The approach followed to build a flexible functional form is to propose a given functional form for a production or cost function, according to the good properties this functional form is supposed to have, and based on an econometric check of its positive concordance with real-world data. A demand or cost share function can then be derived. In the case of a cost function, this is done using Shephard's lemma, which states that, given the fact that a cost or expenditure function is convex, the cost minimizing point of a given good i with price p_i is unique, and can be expressed as follows:

$$D_i = \frac{\partial C}{\partial p_i} \qquad (29)$$

where D_i is the demand for good i, C is the cost function and p_i is the price of good i.

The main concern with these functions is their compliance with the general constraints presented above. The flexible functional forms generally respect by construction the condition of derivability, a further condition of monotonicity for the cost function (that is, the cost function C is increasing in total output Y), and the homogeneity condition. The concavity condition, however, is much more difficult to obtain, and is often only verified in a small price region (Caves and Christensen 1980, Barnett and Lee 1985, Considine 1989b, Terrell 1996).

We present here in detail two flexible functional forms among those most commonly used for the assessment of the impact of prices on energy demand, according to the literature. The translog function is perhaps the most commonly used (Yi 2000), while the linear logit model is one of the last functions to have appeared. The latter has already been tested econometrically by many authors, and seems to give better performance than the translog form, as we explain below.

The translog model

The transcendental logarithmic production frontier (Christensen, Jorgenson and Lau 1972), in short translog, was historically one of the first flexible functional forms to appear. The translog is based on the following cost function (Urga and Walters 2003):

$$\ln C_T = \alpha_0 + \sum_{i=1}^{n} \alpha_i \ln P_{iT} + \alpha_Y \ln Y_T + \alpha_T T + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} \alpha_{ij} \ln P_{iT} \ln P_{jT} + \sum_{i=1}^{n} \alpha_{iY} \ln P_{iT} \ln Y_T + \sum_{i=1}^{n} \alpha_{iT} T \ln P_{iT} + \frac{1}{2}\alpha_{YY} \ln Y_T \ln Y_T + \alpha_{YT} T \ln Y_T + \frac{1}{2}\alpha_{TT} T^2 \quad (30)$$

where the αs are parameters, C_T and Y_T are the cost and output level at time T, and P_{iT} is the price of production factor i at time T. This cost function is a long-run description of the state of the economy and includes the effect of a biased technical change among technologies. The following conditions on the parameters are necessary and sufficient to ensure that C_T is linearly homogeneous:

$$\sum_{i=1}^{n} \alpha_i = 1; \ \forall i, \ \sum_{j=1}^{n} \alpha_{ij} = 0; \ \sum_{i=1}^{n} \alpha_{iY} = 0; \ \sum_{i=1}^{n} \alpha_{iT} = 0 \quad (31)$$

The demand is expressed in the form of cost shares that can be derived using Shephard's lemma:

$$\forall i, \ S_{iT} = \alpha_i + \sum_{j=1}^{n} \alpha_{ij} \ln P_{jT} + \alpha_{iY} \ln Y_T + \alpha_{iT} T \quad (32)$$

where S_{iT} is the cost share of factor i at time T. As we can see, the translog form has the advantage that the sign and level of the parameters can be easily interpreted: for example, the parameters α_{ij} account for the influence of the own price and the other prices on the cost share of energy source i, and should be negative if i=j and positive otherwise. The Allen elasticity of substitution can then be calculated as (Urga and Walters 2003):

$$AES_{ij} = \frac{\alpha_{ij} + S_i S_j}{S_i S_j} \ \text{for} \ i \neq j; \ AES_{ii} = \frac{\alpha_{ii} - S_i + S_i^2}{S_i^2} \quad (33)$$

As we can see, here the elasticity of substitution between factors i and j is not constant and depends on the cost share of factors i and j.

Empirical assessment and limits

As well as the translog model, other forms, such as for example the linear logit model, have been used as the basis for much empirical work in the field of substitution between energy sources, also called interfuel substitution. The translog model was used by a large number of authors from the 1970s to the 1990s (see for example Griffin 1977, Mittelstädt 1983, Hoeller and Coppel 1992, Jones 1995, Renou-Maissant 1998, Urga and Walters 2003); the linear logit model, originally introduced by Considine and Mount (1984), began to be widely used in the 1990s, often in comparison with the translog model (Considine and Mount 1984, Considine 1989a, Jones 1995, Urga and Walters 2003). These works generally assess substitution between three or four types of fuel (coal, oil, natural gas, and often electricity). The assessment can be done using a static or dynamic version of the models. The static versions assume the long-term equilibrium is reached over a one-year period or use panel data, whereas the dynamic versions are obtained by adding a lagged cost share (translog) or quantity ratio (linear logit) term.

Regarding the most popular model for interfuel substitution estimation, namely the translog model, it can be concluded from the empirical work conducted that the estimated parameters often violate basic properties dictated by theories such as concavity (Terrell 1996) or even more basically the sign of the elasticities (Considine 1989a) or the fact that long-run elasticity estimates should be greater than short-run elasticity estimates (Jones 1995). As well, in certain cases the translog model can generate negative share predictions (Lutton and LeBlanc 1984, cited by Considine 1990). As we have already seen, this is a very common problem for all the flexible functional forms (Terrell 1996, Yi 2000).

Several authors have compared the different functional forms, from a theoretical (monotonicity, concavity) or empirical point of view, and concluded that each flexible functional form behaved well over a given price region that depends on the functional form used. Thus a given form should be chosen for each application; for example, the choice between the translog and generalized Leontief forms depends on the degree of variation in the price and income variables and the expected magnitude of the Allen elasticity of substitution (Caves and Christensen 1980). Diewert and Wales (1987) also proposed different remedies to the concavity violations. However, Guilkey, Lovell and Sickles (1983) concluded that the translog form was preferable to other forms such as generalized Leontief or Box-Cox. Regarding the comparison with the linear logit form, Jones (1995) and Urga and Walters (2003) found that the linear logit form gave better empirical results than the translog form in terms of monotonicity, symmetry and concavity, especially in the case of unstable cost shares and/or factor prices.

CONCLUSION OF OVERVIEW

To conclude our review of the state of the art, the field of production, cost and demand functions can be divided into two groups: the CES production functions and their variants, and the flexible functional forms. The functions in the first group impose very strict conditions on the elasticity of substitution or at least the hierarchy between production factors, and thus seem not to be adapted to model the interfuel substitution effects between four or more energy sources. A wide range of functional forms can be found in the second group of functions, and the debate is still open regarding the advantages and disadvantages of each functional form.

The linear logit functional form is used, for example, by the Imaclim model to model the residential end-use energy mix (Crassous, Sassi, Hourcade et al. 2006, Crassous, Sassi, Hourcade, Waisman et al. 2006). However, the recursive estimation technique for the linear logit model (Jones 1995) makes its estimation rather complicated. The implementation of such a complex model also requires a large amount of data, while we only have 19 observations to estimate the parameters of our model. Furthermore, the translog as well as the linear logit demand models produce cost shares (or cost shares ratios for the logit model) as an output: this is not the most obvious presentation one could expect for the assessment of the impact of a carbon tax, where the final aim is to estimate carbon dioxide emissions. For example, a share in the total energy demand seems to be easier to interpret.

A Simple Model for Energy Demand

PRESENTATION OF THE MODEL

In order to build our model, we started from the CES demand function presented above, and compared our final model with the translog model also presented. These forms appeared us to be at the same time theoretically justified and simple to interpret.

The CES demand function of Equation (25) can be re-written for energy source *i* in logarithmic form as follows (logarithmic variables are written in lowercase):

$$x_i - x^m = -\eta\left(p_i - p\right) \qquad (34)$$

where x_i is the logarithm of the level of input of factor *i*, x^m the logarithm of X^m defined in Equation (26), η the elasticity of substitution, p_i the logarithm of the price of factor *i*, and *p* the logarithm of the CES dual price defined in Equation (24). The term on the left side of the equation can be interpreted as the demand for energy *i*, compared to a reference term, stated in Equation (26). This fraction depends (on the right side of the equation) on the elasticity of substitution, multiplied by the price of energy *i*, compared to a mean price, the CES dual price. The latter, given in Equation (24), is the generalized mean of the different energy prices with exponent the constant (and equal above all energy sources) elasticity of substitution η. We can adapt this model to our particular needs as follows.

First, since the x^m term is difficult to interpret, we replace the ratio $x_i - x^m$ used as a dependent variable through an energy share, expressed in energy unit: $e_i - e_{tot}$, where $e_i = \ln(E_i)$ is the logarithm of demand E_i for energy *i* expressed in energy unit (typically in Mtoe), and $e_{tot} = \ln(E_{tot})$ is the logarithm of the total energy demand E_{tot}. This recalls the translog form, which uses cost shares.

There are many long-term, exogenous effects that affect the energy mix: for example, demand for coal has been sinking for years due to more severe regulations on emissions of pollutants, while demand for electricity is increasing. In order to account for these effects, and following Urga and Walters (2003), we add a quadratic trend term $A_i + B_i.T + C_i.T^2$ to the right part of the equation. This term will also absorb the change in the initial model specification made by our replacement of the mean term x^m by the total demand term e_{tot}.

Regarding the right side of the equation, the expression of the CES dual price *P* uses a constant and equal elasticity of substitution. We would like to allow for variable and unequal elasticities of substitution: this leads in a first approximation to a change in the price index. To account for this, we replace the CES dual price *P* with an arithmetic mean of the prices of the various energy sources. This creates a new price index whose coefficients have to be econometrically estimated; the right part of the equation becomes (the subscript T indicates what varies with time):

$$A_i + B_i.T + C_i.T^2 - \eta_i\left(P_{iT} - \sum_{\substack{j=1 \\ j\neq i}}^{n} \rho_j P_{jT}\right) \qquad (35)$$

Finally, and following Jones (1995) and Urga and Walters (2003), we add a lagged term $\beta_i(e_{i(T-1)} - e_{tot(T-1)})$ to take into account the fact that the whole adaptation of demand to a change in price cannot occur in one year. Having rearranged the terms of the price index, the final equation expressing the share of energy demand for energy *i* reads:

$$\underbrace{e_{iT} - e_{totT}}_{Energy\ share} = \underbrace{A_i + B_iT + C_iT^2}_{Time\ trend} + \underbrace{\sum_{j=1}^{n} \alpha_{ij} p_{jT}}_{Influence\ of\ prices} + \underbrace{\beta_i\left(e_{i(T-1)} - e_{tot(T-1)}\right)}_{Lagged\ term} + \varepsilon_{iT} \qquad (36)$$

where A_i, B_i, C_i, α_{ij}, β_i are the parameters to be estimated, and ε_{iT} is the residual term. To ensure homogeneity of degree 0 for the energy share function, we further impose the following constraint on the coefficients:

$$\forall i,\ \sum_{j=1}^{n} \alpha_{ij} = 0 \qquad (37)$$

The final model is very close to the translog form, except for the fact that the dependant variable is an energy share instead of a cost share. This can be interpreted through the fact that energy is considered here in a first approximation as a first-necessity good. Hence it is more appropriate to model it in terms of a share of a total relatively stable energy need instead of a share of a total available expense. Moreover, if needed one can easily estimate the global elasticity of energy demand to the mean weighted price of energy, and combine this outcome with our model to assess the effects of price on energy consumption, including energy efficiency efforts.

Let us calculate the short-term cross-price elasticity of the energy share for fuel *i* in our model, following definition (5):

$$\eta_{ij}^{ST} = \frac{\partial \ln(E_i / E_{tot})}{\partial \ln P_j} = \alpha_{ij} \qquad (38)$$

The long-term cross-price elasticity corresponds to the following long-term equation:

$$(1-\beta_i)(e_i - e_{tot}) = \sum_{j=1}^{n} \alpha_{ij} p_j \qquad (39)$$

Thus we have the following expression for the long-term cross-price elasticity of the energy share for fuel *i*:

$$\eta_{ij}^{LT} = \frac{\partial \ln(E_i / E_{tot})}{\partial \ln P_j} = \frac{\alpha_{ij}}{1-\beta_i} \qquad (40)$$

Some authors impose further constraints on the parameters. For example, the translog model is sometimes used with an imposed symmetry in the elasticities (that is, for each iand j, $\alpha_{ij} = \alpha_{ji}$), see e.g. Hoeller and Coppel (1992) and Urga and Walters (2003). Jones (1995) imposes a common speed of adjustment (lagged term coefficient) for all the energy sources, that is, for each *i*, $\beta_i = \beta_0$. We preferred to impose as few constraints as possible in order to account for the complexity of the real economy.

DATA

Data for yearly energy consumption from 1986 to 2004 in French industry (19 observations) is provided by the CEREN (Centre for study and research on energy), a French research group owned by the ADEME and the main French actors in electricity and gas production and transport. In the original database, energy consumption data is expressed in tonnes of oil equivalent (toe) and prices are expressed in euros. Data provided by the CEREN is segmented using the French NCE 2003 classification; the secondary sector of the economy corresponds to sectors E12 to E38 of the classification, which were aggregated for the purpose of our study. Among the 10 categories of energy sources provided by the CEREN, we selected the five most representative ones: coal, natural gas, heavy fuel oil, heating oil, and electricity. All the other categories represent less than 5% of total energy consumption.

We paid particular attention to the question of the fuels with double usage or non-energy use. This includes, for example, natural gas used for the synthesis of base chemicals or coke used in blast furnaces. Since there is generally no substitute for fuel used as a raw material, or even for double usages, this can lead to inaccurate estimates for the elasticity of energy usages of fuel. Jones (1995) simply removes the consumption of fuels used for non-energy purposes; he pointed out that this had a great impact on the elasticities and the rate of dynamic adjustment. We propose to remove the entire consumption of the industrial sub-sectors concerned, both for energy and non-energy use. This can be justified by the fact that a firm that initially uses a given fuel both for energy production and for non-energy use would shift energy sources less easily than another firm, since both usages (energy and non-energy) may be combined in the production process, for economic or practical reasons. Moreover, fuel used for double usage should be exonerated from a possible carbon tax in France: thus in our context we do not wish to include these sectors in our estimates of fuel demand. The sectors removed from energy-use data are the following, expressed using the French NCE 2003 classification: ironworks and first transformation of steel (E16, E17), metallurgy and first transformation of nonferrous metals (E18), manufacture of plastic, synthetic rubber and other elastomers (E25), other base chemicals industries (E26), the synthetic textiles industry (E27), and the parachemistry and pharmaceutical industry (E28).

Annual price data for each of these energy sources is provided by Enerdata, a French energy consulting and information services company, and originally comes from the International Energy Agency and the French "Observatoire de l'Energie". The prices correspond to end-use prices for a representative mean annual consumption and are expressed as real prices per MWh.

ESTIMATION AND RESULTS

We estimated the following equation, obtained by replacing the parameter α_{ii} in (36) using (37):

$$e_{iT} - e_{totT} = A_i + B_i T + C_i T^2 + \sum_{\substack{j=1 \\ j\neq i}}^{n} \alpha_{ij} p_{jT} - \left(\sum_{\substack{j=1 \\ j\neq i}}^{n} \alpha_{ij} \right) p_{iT} + \beta_i \left(e_{i(T-1)} - e_{tot(T-1)} \right) + \varepsilon_{iT} \qquad (41)$$

We used the Ordinary Least Squares (OLS) method for our econometrical estimation. When the parameters were estimated for the whole model including all 5 different energy prices, there were always several non-significant price coefficients (according to Student's t-statistic, see Appendix). The non-significant coefficients were removed from the model for the final estimation presented here, in Table 1. The short-term and long-term cross-price elasticities of energy shares were also computed and are given in Table 2.

The estimated equations show rather good values for the coefficient of determination R^2 (it is not too surprising since the lagged endogenous variable and trend variables are included in the set of explanatory variables) and Durbin's h-statistic (autocorrelation in the residuals). The Augmented Dickey-Fuller (ADF) test did not reject at the 95 per cent level the null hypothesis of a unit root for the logarithm of the energy share of coal, heavy fuel oil, heating oil and electricity.

Table 1. Estimated parameters after removal of non-significant ones. Each column stands for one Equation (41), for a given energy source *i*. The figures in parentheses are the probabilities associated with the t-statistics; the own-price parameters have been recomputed using Equation (37).

Param.	Descr.	Coal share	Heavy fuel oil share	Heating oil share	Natural gas share	Electricity share
A_i	Constant	0.436	-1.419		-0.267	-0.744
		(0.46)	(0.00)		(0.37)	(0.05)
B_i	Linear trend term	-0.259		-0.189	0.00316	0.0193
		(0.00)		(0.00)	(0.51)	(0.09)
C_i	Quadratic trend term	0.00462	-9.80E-4	0.00272		-2.68E-4
		(0.00)	(0.00)	(0.00)		(0.21)
α_{iC}	Coal price	-0.674				
α_{iH}	Heavy fuel oil price		-0.102			
α_{iL}	Heating oil price	0.441		-0.0827		
		(0.01)				
α_{iG}	Natural gas price	-0.521			-0.0520	0.0510
		(0.04)				(0.16)
α_{iE}	Electricity price	0.754	0.102	0.0827	0.0520	-0.510
		(0.05)	(0.00)	(0.41)	(0.17)	
β_i	Lagged term	0.350	0.0638	0.118	0.844	0.506
		(0.08)	(0.71)	(0.66)	(0.00)	(0.02)
R^2		0.993	0.997	0.977	0.983	0.985
Durbin's h		-1.10	-0.787	0.437	-0.898	0.306
ADF test statistic (probability)[1]		-1.56 (0.48)	0.765 (0.99)	-2.40 (0.16)	-3.12 (0.04)	-2.47 (0.14)

[1] *ADF test conducted on the energy shares. Probabilities are calculated for 20 observations and may not be accurate for a sample size of 18.*

Table 2. Short-term (ST) and long-term (LT) cross-price elasticities for energy shares, computed using Equations (38) and (40)

		Coal	Heavy fuel oil	Heating oil	Natural gas	Electricity
Coal	ST	-0,674				
	LT	-1,04				
Heavy fuel oil	ST		-0.102			
	LT		-0.109			
Heating oil	ST	0.441		-0.0827		
	LT	0.678		-0.0934		
Natural gas	ST	-0.521			-0.0520	0.0510
	LT	-0.801			-0.333	0.103
Electricity	ST	0.754	0.102	0.0827	0.0520	-0.0510
	LT	1.16	0.109	0.0927	0.334	-0.103

The values of all the coefficients are consistent with intuition and theory, except for the negative effect of an increase in the price of natural gas on the consumption of coal. This effect could be explained by the fact that one of the by-products of coke production from coal is methane, which generally goes into the natural gas transmission network, even if this is probably not a sufficient explanation.

The results show up that own-price elasticity is by far the greatest for coal, both in the short term and in the long term, while it is weakest for natural gas and electricity. In the long run however, the demand for natural gas is much more sensitive to price than that for oil or electricity.

The value of the lagged terms also shows a very fast speed of adjustment for heavy fuel oil and heating oil (around 90% of the adjustment is done in the first year), while the installations using gas as an energy source are the slowest to adapt to a change in prices.

The significant substitution effects between coal and almost every other energy source (electricity being the preferred alternative energy source, followed by natural gas and light fuel) could be explained by the general decrease in the consumption of coal because of its negative image and increasing environmental regulation, which is often hostile to coal. Thus coal, which used to be a very important energy source for many uses, is replaced by various energy sources depending on the specific usage. The other sources, however, are mainly substitutable with electricity, which is perhaps the most universal energy source.

These results also show that, except for coal, the effect of energy price on energy demand is probably mainly attributable to the improvements in energy efficiency resulting in less energy demand. The decrease in the prices of the other energy sources seems not to be a sufficient incentive to change the energy source of production processes, at least for variations in prices of the order of magnitude of those occurring over the time period used to estimate the parameters of the model.

Conclusion and Further Improvements

In conclusion, in our attempt to assess the impact of a carbon tax on emissions of CO_2, after an overview of the existing production, cost and demand functions with their respective characteristics, we built an energy demand model adapted to our particular needs. This model is based on several well-known models, but allows us to assess the direct effect of the prices of the different energy sources on the substitution effects between energy sources. The econometric estimation of the parameters of the model gives results that can be interpreted both economically and technically.

The main conclusion that can be drawn from the econometric study is that the substitution effects between energy sources due to changes in prices are not as significant as might be expected, at least for variations in prices of the order of magnitude of those occurring over the time period used to estimate the parameters of the model, after having removed the effect of the trend (our model is not designed for greater variations anyway). In particular, the only effect that proves to be statistically significant in our model is the substitution with electricity, which is perhaps the most general form of energy. Coal is a special case, probably due to the fact that on the one hand the overall consumption of coal is decreasing and on the other hand coal used to be an all-purpose energy source like electricity.

Since the main effect of a carbon tax on energy consumption seems to be an incentive to consume less of the current energy source, rather than shifting to other energy sources, this raises the question of the choice of our model. Perhaps a model including the level of energy demand would be more appropriate in order to take improvements in energy efficiency into account more effectively. A more sophisticated method for econometrical evaluation could also be tried, such as for example a Kalman filter to take exogenous effects into account more effectively (Reynès 2006). The main obstacle to such methods was the lack of observations for the evaluation: this could be avoided by multiplying the number of observations using panel data drawn from the different sectors of French industry.

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Appendix

Table 3. Estimated parameters without removal of non-significant ones. Each column stands for one Equation (41), for a given energy source *i*. The figures in parentheses are the probabilities associated with the t-statistics.

Param.	Descr.	Coal share	Heavy fuel oil share	Heating oil share	Natural gas share	Electricity share
A_i	Constant	0.447	-1.52	-0.355	-1.605	-0.886
		(0.48)	(0.00)	(0.65)	(0.06)	(0.23)
B_i	Linear trend term	-0.257	-7.26E-05	-0.189	0.0695	0.0203
		(0.00)	(1.00)	(0.06)	(0.07)	(0.22)
C_i	Quadratic trend term	0.00460	-0.00111	0.00258	-0.00108	-2.60E-04
		(0.00)	(0.03)	(0.11)	(0.08)	(0.30)
α_{iC}	Coal price		-0.0241	-0.00538	0.139	-0.0610
			(0.90)	(0.99)	(0.51)	(0.74)
α_{iH}	Heavy fuel oil price	0.0216		-0.168	0.0120	0.0200
		(0.87)		(0.33)	(0.78)	(0.58)
α_{iL}	Heating oil price	0.427	-0.0935		-0.00787	-0.0564
		(0.02)	(-0.21)		(0.89)	(0.26)
α_{iG}	Natural gas price	-0.535	0.0317	0.120		0.0890
		(0.05)	(0.78)	(0.71)		(0.24)
α_{iE}	Electricity price	0.761	0.147	0.0212	-0.0992	
		(0.06)	(0.36)	(0.96)	(0.56)	
β_i	Lagged term	0.369	-0.0380	-0.012	0.240	0.423
		(0.13)	(0.87)	(0.97)	(0.53)	(0.02)
R^2		0.993	0.997	0.980	0.988	0.987
Durbin's h		-1.16	-0.189	0.303	-0.332	0.00485